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A Set-Theoretical Characterization of the Torus

By

Tadashi TANAKA

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Introduction

We have known the set-theoretical properties which characterize the closed 2-cells and the 2-spheres among locally connected, compact, metric continua. In this paper we shall consider the set-theoretical properties which characterize the torus.

Most of the terminologies and notations are due to G. T. Whyburn's book [1].[1)]

§1. Statement of Theorem

For the convenience of the statement of Theorem, we shall give the following definitions.

DEFINITION. A set M will be said to be *locally connected* at a point p ($p \in M$ or $p \notin M$) if for any $\varepsilon > 0$ there exists $\delta > 0$ such that every two points x and y of M whose distances from p are less than δ lie together in a connected subset of M of diameter less than ε.

DEFINITION. Let M be a locally connected set and N a subset of M which separates a sufficiently small region containing a point of N. Let $X = ab$ be a spanning arc of N in M, U a sufficiently small region in M containing a and V the component of $U - N$ which contains a neighborhood of a in $\langle X \rangle$. We shall say that a *spanning arc* Y *of* N *covers the end point* a *of* X if Y is an arc such that $\langle Y \rangle$ lies in V and its end points are the end points of an arc in N which has a as an interior point.

In the following sections, we shall prove

THEOREM. *In order that a locally connected, compact, metric continuum* T *be a torus it is necessary and sufficient that* $T = J_1 + J_2 + R$ *where*

(a) J_i*'s* $(i=1,2)$ *are simple closed curves such that* $J_1 \cdot J_2 = p$ *is a single point, and* R $(= T - (J_1 + J_2))$ *is connected;*

(b) J_i *separates irreducibly a sufficiently small connected neighborhood* R_i *of* J_i *into exactly two regions* R_i^1 *and* R_i^2, *being locally connected at any point of* J_i;

(c) *if* $X = ab$ *is any spanning arc of* $J_1 + J_2$ *in* T, *then* $\langle X \rangle$ *separates* R *irreducibly, and moreover if* Y *is a spanning arc of* $J_1 + J_2$ *in* T *which*

1) Number in brackets refer to the references at the end of the paper.