JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 21, NO. 2, JULY, 1957

On a Locally Convex Space Introduced By J.S.E Silva

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(Received April 30, 1957)

Introduction

In the recent paper [8], J.S.E Silva has given a detailed discussion on the (LN^*) space, a certain type of an inductive limit of normed spaces, in view of applications to the theory of analytic functions and to the theory of distributions. His way of proving the properties of (LN^*) space, which are requisite to applications, is based rather on the consideration of the spectre defining the space than on the direct investigation of the (LN^*) space itself as a locally convex space.

In the present paper, we give certain necessary and sufficient conditions for a locally convex space to be an (LN^*). And basing on this characterization, though some of the results are due to Silva, we prove certain fundamental properties of (LN^*) spaces. Among other things it is shown that a closed subspace of an (LN^*) is an (LN^*), that an inductive limit of a countable number of (LN^*) is an (LN^*), and that the strong dual of a Schwartz (LF) space is nothing but a projective limit of a sequence of (LN^*) of certain type.

In most cases of this paper, notions and terminologies of Bourbaki [1] are used without explicit references.

§1. Preliminaries

Let E be a locally convex space over the complex numbers. If $A(\subset E)$ is a bounded closed disk (=convex circled subset), E_A denotes the normed space generated by A with the norm $\|x\|_A = \inf_{x\in\lambda A} |\lambda|$, $x \in E_A$. E is a (DF) *space* [5] if

(1) E has a fundamental sequence of bounded sets;

(2) if the intersection $\bigcap_k U_k$ of a sequence of closed convex circled neighbourhoods U_k of the origin absorbs every bounded set, $\bigcap_k U_k$ is a neighbourhood. E is said to satisfy the *strict Mackey condition* if for every bounded set $A \subset E$ there exists a bounded closed disk $B \supset A$ such that the topology on A induced by E is identical with the topology on A induced by E_B. E is called a *Schwartz space* [5] if