JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 21, NO. 2, JULY, 1957

On Splitting of Valuations in Extensions of Local Domains

By

Hazimu SATÔ

(Received April 25, 1957)

Introduction. In their paper [1], S. Abhyankar and O. Zariski proved a theorem concerning splitting of valuations in extensions of local domains, and in the subsequent paper [2], Abhyankar generalized the theorem in the following form:

Let (R, M) be a local domain of dimension >1, such that either (a) R admits a nucleus, or (b) R is regular and has the same characteristic as its residue field. Let K be the quotient field of R and K^* a finite separable extension of K. Then there exist infinitely many real discrete valuations v of K with the following two properties: (1) v has center M in R, and (2) any K^*-extension v^* of v has degree 1 over v, or equivalently, v has exactly $[K^* : K]$ distinct extensions to K^*.

The purpose of this paper is to generalize this result by removing any condition imposed upon R, and to give a considerably simpler proof than the original ones. We state this generalization as our

Theorem. *Let (R, M) be a (general) local domain of dimension $d>1$ and let K^* be a finite separable algebraic extension of the quotient field K of R. Then there exist infinitely many real discrete valuations v of K having the following two properties:* (1) *v has center M in R, and* (2) *any K^*-extension v^* of v has degree* 1 *over v.*

Following Abhyankar closely, first we prove, in §1, that there exist infinitely many real discrete valuations v of K such that v has center M in R and v has more than one extension to K^* $(K \neq K^*)$, and secondly, with the aid of this result, our theorem will be proved in §2.

The writer wishes to express his hearty thanks to Mr. M. Yoshida for his valuable suggestions and kind criticism, and also to M. Sakuma for his kind advice.

We shall use the following notations: If v is a valuation of a field K, then R_v will denote the valuation ring of v and M_v will denote the valuation ideal of v. If R is a local ring with the maximal ideal M, then we shall express this by saying, "(R, M) is a local ring".

§1. Splitting of valuations.

We shall prove the following: