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On the Role of Hamiltonians in the Relativistic Dynamics referred to the New Fundamental Group of Transformations

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§ 1. Introduction.

In the usual form of non-relativistic dynamics, the description of the dynamical system is done by the so-called instant form, and the non-trivial change of the system is caused by the *one* Hamiltonian. This corresponds to the fact that, in the non-relativistic dynamics, the transformation connecting two equivalent observers is Galilean transformations which *leave the instant of time $t=0$ invariant, and only the translation of time changes the instant.* On the other hand, in the relativistic dynamics, two equivalent coordinate systems are related by the special Lorentz transformations by which the instant of time $t=0$ is not remained invariant. Therefore, it is necessary to introduce extra Hamiltonians other than that of non-relativistic dynamics. Among the fundamental quantities $P_\mu, M_{\mu\nu}$ which satisfy the following P.b. (Poisson bracket) relations:

$$\left.\begin{aligned} &[P_\mu, P_\nu]=0, \quad [M_{\mu\nu}, P_\rho]=-g_{\mu\rho}P_\nu+g_{\nu\rho}P_\mu \quad (\mu, \nu=1, \cdots, 4) \\ &[M_{\mu\nu}, M_{\rho\sigma}]=-g_{\mu\rho}M_{\nu\sigma}+g_{\nu\rho}M_{\mu\sigma}-g_{\mu\sigma}M_{\rho\nu}+g_{\nu\sigma}M_{\rho\mu} \end{aligned}\right\} \tag{1.1}$$

with $\quad -g_{11}=-g_{22}=-g_{33}=g_{44}=1, \quad g_{\mu\nu}=0$ for $\mu \neq \nu$,

and leave the instant invariant, Dirac [1]† called the following four quantities:

$$P_4, \ M_{4i} \qquad (i=1, 2, 3) \tag{1.2}$$

as Hamiltonians. The discussion about what roles the extra Hamiltonians M_{4i} other than the usual P_4 play is not yet dealt with at the present.

On the other hand, in the instant form, we are obliged to consider that the difference of the descriptions of a dynamical variable in two coordinate systems related by the special Lorentz transformations is caused by Hamiltonians. Therefore, the relativistic instant form has not a parallelism with non-relativistic dynamics. Thus, we are led to the following problem: Can we have then the new dynamical form having this parallelism? We investigated this problem in the previous paper [2], and attempted to construct the new dynamical form satisfying the requirement that the descriptions of a dynamical variable by two observers one of which

† Numbers in brackets refer to the references at the end of the paper.