JOURNAL OF SCIENCE OF THE HIROSHIMA UNIVERSITY, SER. A, VOL. 20, NO. 1, MAY, 1956

Spin-orbit Interaction Energy of an Electron based on the New Fundamental Group of Transformations

By

Takashi SHIBATA and Toshiei KIMURA

(Received Feb. 3, 1956)

§ 1. Introduction

Spin-orbit interaction energy of an electron was first introduced by Uhlenbeck and Goudsmit [1]*) showing how the idea of spin of an electron can be used to explain the anomalous Zeeman effect. However, the assumption they had to make seemed to lead to the value twice as large as the experimental value. It was due to the fact that the calculation was referred to the observer moving instantaneously with the same velocity as that of an electron. But, in fact, the velocity of an electron in atom changes successively for the observer fixed to the nucleus. Thereupon, taking it into account, L. H. Thomas [2] and J. Frenkel [3] calculated the value of spin-orbit interaction energy. Their calculations led to the same value as observed. In this way, it was shown that Uhlenbeck and Goudsmit's assumptions really led to the correct doublet separation at the same time as the anomalous Zeeman effect when the problem was treated by the quantum mechanics [4]. Nowadays, the effect of the spin-orbit interaction on the energy levels of hydrogen is deduced from the Dirac's theory of electron. However, in this paper, we shall describe our theory comparing it with that of L. H. Thomas.

The main fact used by L. H. Thomas in obtaining the correct result is that the combination of two "Lorentz transformations without rotation" in general is not of the same form but is equivalent to a Lorentz transformation with a rotation. The "Lorentz transformations without rotation" are defined by the following equations [2, 5, 6, 7]:

$$\left.\begin{aligned} x'^i &= x^i + u^i\left[\frac{(ux)}{u^2}\left\{\frac{1}{\sqrt{1-u^2/c^2}}-1\right\}-\frac{t}{\sqrt{1-u^2/c^2}}\right] \quad (i=1,2,3) \\ t' &= [t-(ux)/c^2]/\sqrt{1-u^2/c^2} \end{aligned}\right\} \tag{1.1}$$

where x^i, t and x'^i, t' denote the space-time coordinates in the systems K and K', the components of the uniform velocity of K' relative to K being u^h $(h=1, 2, 3)$. The round parenthesis of u and x: (ux) denotes the inner product of u^i and x^i. The above-mentioned fact means that the trans-

*) Numbers in brackets refer to the references at the end of the paper.