J. SCI. HIROSHIMA UNIV. SER. A
VOL. 24, NO. 3, DECEMBER, 1960

On partial Summability aud Convolutions in the Theory of Vector Valued Distributions

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(Received September 20, 1960)

In his theory of vecter valued distributions ([11], [12]) L. Schwartz introduced the concept of partial summability of a kernel distribution, which makes it possible to give the precise meaning to the Fourier integral written formally by $\int e^{-2\pi i \dot{x} y} T(y)\, dy$ in his work [8]. For any tempered distribution T, the distribution $e^{-2\pi i \dot{x}\dot{y}}\ T(\dot{y})$ is partially summable with respect to y and the partial integral $\int e^{-2\pi i \dot{x} y}\ T(y) dy$ is the Fourier transform of T. We show in Sec. 1 below that, for a distribution T, if $e^{-2\pi i \dot{x}\dot{y}}\ T(\dot{y})$ is partially summable with respect to y, T must be tempered. Consequently the set of distributions T for which $e^{-2\pi i \dot{x}\dot{y}}\ T(\dot{y})$ is partially summable with respect to y is exactly $\mathcal{S}'$, the space of tempered distributions. We show that the same is also true for vector valued distributions.

One of the present authors proved [13] that various definitions concerning the convolution of two distributions which are available in the literature are equivalent to each other. The results established there will be generalized for kernel distributions $K(\dot{x}, \dot{y})$ on $R^n \times R^n$ [see Sec. 2]: $K(\dot{x}-\dot{y}, \dot{y})$ is partially summable with respect to y if and only if $\varphi(\dot{x}+\dot{y})\ K(\dot{x}, \dot{y})$ is summable for any $\varphi \in \mathcal{D}$. The convolution $\overset{*}{K}$ is defined by $<\overset{*}{K}, \varphi> = \iint \varphi(x+y)\ K(x, y)\, dxdy$, $\varphi \in \mathcal{D}$ or by $\int K(\dot{x}-y, y)\, dy$. The analogous considerations on $\mathcal{S}'$-convolutions are given. The concept of $\mathcal{S}'$-convolution of two distributions was first introduced in [5] and its further investigation was carried out in [13]. We introduce the space (noted by $\mathcal{O}'_{x,y}$) of kernel distributions for which the convolution is defined. If we $\mathcal{O}'_{x,y}$ take the topology introduced in a natural manner, then $\mathcal{O}'_{x,y}$ will be a permitted, ultra-bornological, complete space of distributions on $R^n \times R^n$.

Finally Sec. 3 is concerned with the convolution defined by starting with the tensor product of vector valued distributions. The results obtained for ordinary distributions [13] will be extended to vector valued distributions, especially we show that $\vec{S}(\dot{x}-\dot{y}) \otimes\otimes_{\iota} \vec{T}(\dot{y})$ is partially summable with respect to y if and only if $\varphi(\dot{x}+\dot{y})\ (\vec{S}_x \otimes\otimes_{\iota} \vec{T}_y)$ is summable. We believe that this result will be of use for further investigation of Schwartz' theory of the convolutions of vector valued distributions.