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On $\mathfrak{m}$*-adic Differentials*

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Introduction

In our previous paper one of the authors developed the theory of differentials in commutative rings. In that paper he started with the definitions given in [1][1] and applied the theory of differentials to the characterization of geometric regular local rings, and other problems. But when he was going to apply the theory to a larger class of local rings he encountered serious embarrassment. The situation may be well described if we take up the following example. Let A be the ring of the analytic functions in n complex variations $z_1, \cdots z_n$, which are regular in some neighborhood of the origin. According to the definition given in [1], the module of differentials in A (over the constant field) contains infinitely many linearly independent differentials. On the other hand if f is an element of A, the differential df can be written uniquely in the form $df = \frac{\partial f}{\partial z_1} dz_1 + \cdots + \frac{\partial f}{\partial z_n} dz_n$, i.e. there exist only n-analytically independent differentials $dz_1, \cdots, dz_m$ in A. This example means that the former theory is not adequate as a theory of differentials since it cannot cover the analytic case. Now it may be natural to ask whether there exists a good algebraic theory available for such a case. This is the motivation of the present work and the affirmative answer will be presented here. To develop the theory we must begin with $\mathfrak{m}$-adic rings and then the notion of $\mathfrak{m}$-adic differentials will be introduced. Naturally the newly introduced notion contains the old one as a special case. Moreover it will be seen that when we deal with the local rings of points on an algebraic variety, the new theory coincides with the former one. This is the reason why the former theory is useful to the problems in algebraic geometry. In the last paragraph we shall give the characterizations of regular local rings as an application. We hope that this theory will find good applications in the theory of algebroid varieties.

§ 1. Preliminaries.

We shall retain all the notations and terminologies used in [4]. All rings considered in this paper will be commutative containing the unity 1. Let S be an R-algebra with a ring homomorphism $f: R \rightarrow S$ such that $f(1)=1$. We shall denote by $D_R(S)$, *the module of R-differentials* in S, and R-differential

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1) The number in the bracket refer to the bibliography at the end of the paper.