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On the average of some arithmetical functions under a constraint on the sum of digits of squares

Dedicated to my parents Hédi and Wiem for their endless support, with love

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Abstract. Let $q \geq 2$ be an integer and $S_q(n)$ denote the sum of the digits in base q of the positive integer n. We look for an estimate of the average of some multiplicative arithmetical functions defined by sums over divisors d of n satisfying $S_q(d^2) \equiv r \mod m$ for some integers r and m.

1. Introduction

Throughout this paper, we denote by $\mathbf{N}$, $\mathbf{N}_0$, $\mathbf{Z}$, $\mathbf{R}$ and $\mathbf{C}$ the sets of positive integers, non negative integers, integers, real and complex numbers respectively. Given a real number x, $\lfloor x \rfloor$ denotes the greatest integer $\leq x$ and $e(x) = e^{2i\pi x}$. The greatest common divisor of two integers a and b will be denoted by (a, b) and if $a \leq b$ we denote by $[\![a, b]\!]$ the set $\{a, a+1, \dots, b\}$. The number of distinct prime factors of a positive integer n will be denoted $\omega(n)$.

First, we shall introduce the following definition: let $n \in \mathbf{N}_0$ and q be an integer ≥ 2. The sequence $(a_j(n))_{j \in \mathbf{N}_0} \in \{0, 1, \dots, q-1\}^{\mathbf{N}_0}$ is defined to be the unique sequence satisfying

$$n = \sum_{k=0}^{\infty} a_k(n) q^k. \tag{1.1}$$

The right hand side of the expression (1.1) shall be called the *expansion* of n to the *base* q. We shall set

$$S(n) = S_q(n) = \sum_{k=0}^{\infty} a_k(n).$$

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