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The space of geometric limits of abelian subgroups of $\mathrm{PSL}_2(\mathbb{C})$

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Abstract. We give a local description of the topology of the space of all geometric limits of closed abelian subgroups of $\mathrm{PSL}_2(\mathbb{C})$. More precisely, we give geometric descriptions for all possible neighborhoods of a point of this space. Intuition from hyperbolic geometry plays an important role by identifying $\mathrm{PSL}_2(\mathbb{C})$ with the group of isometries of $\mathbb{H}^3$. The tools and ideas developed in the authors' previous paper on one-generator closed subgroups of $\mathrm{PSL}_2(\mathbb{R})$ allow one to reduce this problem to a problem about the geometric limits of certain closed subgroups of $\mathbb{C}$ and $\mathbb{C}^*$.

1. Introduction

The present article was announced in [3], in which the authors, motivated by the desire to understand the closure of the faithful discrete type-preserving $\mathrm{PSL}_2(\mathbb{C})$-representations of the fundamental group of the once-punctured torus, gave a complete description of the closure of the space of one-generator closed subgroups of $\mathrm{PSL}_2(\mathbb{R})$ for the Chabauty topology. See [5] for a general exposition of Chabauty topology; we also included in [3] a mini-History of Chabauty topology and related topics.

In the world of geometric limits of Kleinian groups, a sequence of infinite cyclic groups each of which is generated by one hyperbolic isometry can converge to a subgroup isomorphic to $\mathbb{Z}^2$, whose generators are both parabolic isometries. This fact can be equivalently stated using Chabauty topology, a topology one puts on the space of the closed subgroups of $\mathrm{PSL}_2(\mathbb{C})$. The existence of such a behaviour was first observed by Jorgensen. See [9], Section 5 and [11], Example 9.14 for more detail.

A natural question arising is how to find conditions on a given sequence of groups for the limit group to exist, and to describe this limit group.

In this paper we answer this question for an arbitrary sequence of abelian subgroups of $\mathrm{PSL}_2(\mathbb{C})$ by using an "exhaustion of cases" approach (see Subsection 2.4: Strategy).

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