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Operator theory and the Oka extension theorem

Jim AGLER and John E. McCARTHY

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ABSTRACT. For δ an m-tuple of analytic functions, we define an algebra $H^\infty_{\delta,\mathrm{gen}}$, contained in the bounded analytic functions on the analytic polyhedron $\{\|\delta^l(z)\| < 1, 1 \le l \le m\}$, and prove a representation formula for it. We give conditions whereby every function that is analytic on a neighborhood of $\{\|\delta^l(z)\| \le 1, 1 \le l \le m\}$ is actually in $H^\infty_{\delta,\mathrm{gen}}$. We use this to give a proof of the Oka extension theorem with bounds. We define an $H^\infty_{\delta,\mathrm{gen}}$ functional calculus for operators.

1. Introduction

In [4] a new proof of the Oka Extension Theorem for p-polyhedra was given using operator-theoretic methods. While the proof used both the functional calculus for commuting operator-tuples and the Oka-Weil approximation theorem for analytic functions defined on a neighborhood of p-polyhedra, it had the novel feature that it revealed norms for which extensions were obtained with precise bounds.

In this paper we shall improve upon the results from [4] in a number of ways. First, we shall eliminate our reliance on the functional calculus for general operator-tuples developed by J. Taylor, and in its place, use a simple functional calculus for commuting diagonalizable matrix-tuples together with a technical axiom described in Section 3 of the paper. Secondly, we shall not require the Oka-Weil approximation theorem. Rather, using our technical axiom we shall give an original proof of the stronger Oka Extension Theorem [10] that is entirely elementary in nature. Finally, rather than working on p-polyhedra, we shall work with analytic polyhedra defined in general domains of holomorphy.

Once the program described in the above paragraph is carried out, the door is opened for generalization of Oka and Cartan like results in several complex variables that involve the existence of extensions of holomorphic

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