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Existence and analyticity of solutions to the drift-diffusion equation with critical dissipation

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Abstract. The initial value problem for the drift-diffusion equation arising from a model of semiconductor-devices is studied. The goal in this paper is to derive well-posedness and real analyticity of solutions of the initial value problem for the drift-diffusion equation with its dissipating term $\Lambda = (-\Delta)^{1/2}$. In the preceding works for some associated equations, the case corresponding to this is known as critical. In this case, the drift-diffusion equation with Λ is of elliptic type, so we may not apply the L^p-theory for parabolic partial differential equations used in the case that the dissipating term is $\Lambda^\theta = (-\Delta)^{\theta/2}$ with $1 < \theta \le 2$.

1. Introduction

We consider the following initial value problem for the drift-diffusion equation arising from a model of semiconductors:

$$\begin{cases} \partial_t u + \Lambda^\theta u - \nabla \cdot (u \nabla \psi) = 0, & t > 0,\ x \in \mathbf{R}^n, \\ -\Delta \psi = u, & t > 0,\ x \in \mathbf{R}^n, \\ u(0,x) = u_0(x), & x \in \mathbf{R}^n, \end{cases} \tag{1}$$

where $n \ge 2$, $1 \le \theta \le 2$, $\partial_t = \partial/\partial t$, $\nabla = (\partial_1, \dots, \partial_n)$, $\partial_j = \partial/\partial x_j$ $(j = 1, \dots, n)$, $\Lambda^\theta \varphi = \mathscr{F}^{-1}[|\xi|^\theta \mathscr{F}[\varphi]]$, $\Delta = \sum_{j=1}^n \partial_j^2$, and $u_0 = u_0(x)$ is given real valued initial data. The unknown functions $u = u(t,x)$ and $\psi = \psi(t,x)$ stand for the density of electrons and the potential of electromagnetic-field in a semiconductor, respectively. When $\theta = 2$, the dissipative operator Λ^2 gives the positive Laplacian $-\Delta$. When $1 \le \theta < 2$, the fractional Laplacian Λ^θ involves the jumping-process in the stochastic-process and it gives the suitable dissipation to describe the dynamics of electrons in a semiconductor. This operator yields the anomalous diffusion in dissipative equations. For the basic properties of

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