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C^ℓ-contact and C^ℓ-right equivalences of real semi-quasihomogeneous C^ℓ function germs

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Abstract. In this paper we investigate the C^ℓ versions of contact and right equivalences of real semi-quasihomogeneous C^ℓ function germs, $1 \le \ell \le \infty$. The C^ℓ-right equivalence implies C^ℓ-contact equivalence for any $1 \le \ell \le \infty$ and in this work we show, up to certain conditions, that for semi-quasihomogeneous C^ℓ function germs the converse is also true (Theorem 1). As a consequence, concerning the particular case of quasihomogeneous C^ℓ function germs, we also have a similar result (Corollary 1) which recover a known result of M. Takahashi [14] for $\ell = \infty$. We note that we are considering semi-quasihomogeneous function germs with no additional hypothesis of isolated singularity at zero.

1. Introduction

For any ℓ with $1 \le \ell \le \infty$, two C^ℓ function germs $f, g : (\mathbf{R}^n, 0) \to (\mathbf{R}, 0)$ are:

- C^ℓ*-right equivalent* if there exists a C^ℓ-diffeomorphism germ $h : (\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0)$ such that $g = f \circ h$.
- C^ℓ*-contact equivalent* if there exist a C^ℓ-diffeomorphism germ $h : (\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0)$ and a non-zero C^ℓ function germ $M : (\mathbf{R}^n, 0) \to \mathbf{R}$, with $M(0) \neq 0$, such that $g = M \cdot f \circ h$.

These two equivalence relations are denoted by C^ℓ-$\mathscr{R}$ and C^ℓ-$\mathscr{K}$ equivalences, respectively. Also, when $l = \infty$ we just write $\mathscr{R}$ instead of C^∞-$\mathscr{R}$ and $\mathscr{K}$ instead of C^∞-$\mathscr{K}$, respectively.

It is easy to see that C^ℓ-$\mathscr{R}$-equivalence implies C^ℓ-$\mathscr{K}$-equivalence, but the converse does not hold in general. For instance, if $\ell = \infty$, the germs $f(x) = x^2$ and $g(x) = -x^2$ are $\mathscr{K}$-equivalent but they are not $\mathscr{R}$-equivalent. Hence, it seems an important problem to clarify the relationship between C^ℓ-$\mathscr{R}$ and C^ℓ-$\mathscr{K}$ equivalences. Recently, this subject was studied by some authors when $l = \infty$ and for the class of quasihomogeneous C^∞ function germs (cf.

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