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Oscillation criteria for nonlinear differential systems with general deviating arguments of mixed type

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1. Introduction

In this paper we consider the nonlinear differential system with deviating arguments of the form

$$(S_\lambda) \qquad y_i'(t) = p_i(t) y_{i+1}(h_{i+1}(t)), \qquad i = 1, 2, \dots, n-1,$$

$$y_n' = (-1)^\lambda \sum_{m=1}^{N} a_m(t) f_m(y_1(g_m(t))), \qquad t \geq 0, \qquad n \geq 2, \qquad \lambda \in \{1, 2\},$$

under the following standing assumptions:

- (A_1) p_i: $[0, \infty) \to [0, \infty)$, $(i = 1, 2, \dots, n-1)$ are continuous functions and $\int_0^\infty p_i(t) dt = \infty$, $i = 1, 2, \dots, n-1$;
- (A_2) a_m: $[0, \infty) \to [0, \infty)$, $(m = 1, 2, \dots, N)$ are continuous functions and are not identically zero on any infinite subinterval of $[0, \infty)$;
- (A_3) h_i: $[0, \infty) \to R$, $(i = 2, 3, \dots, n)$ are continuously differentiable functions with $h_i'(t) > 0$ on $[0, \infty)$, and $\lim_{t\to\infty} h_i(t) = \infty$ for $i = 2, 3, \dots, n$;
- (A_4) g_m: $[0, \infty) \to R$ $(m = 1, 2, \dots, N)$ are continuous functions and $\lim_{t\to\infty} g_m(t) = \infty$ for $m = 1, 2, \dots, N$;
- (A_5) f_m: $R \to R$ $(m = 1, 2, \dots, N)$ are continuous functions and $u f_m(u) > 0$ for $u \neq 0$, $m = 1, 2, \dots, N$.

By a proper solution of the system (S_λ) we mean a solution $y = (y_1, y_2, \dots, y_n) \in C^1[[T_y, \infty), R]$ which satisfies (S_λ) for all sufficiently large t, and $\sup\left\{\sum_{i=1}^n |y_i(t)|; t \geq T\right\} > 0$ for any $T \geq T_y$. We make the standing hypothesis that the system (S_λ) does possess proper solutions.

A proper solution of (S_λ) is called oscillatory if each of its component has arbitrarily large zeros. A proper solution of (S_λ) is called nonoscillatory (weakly nonoscillatory) on $[T_y, \infty)$ if each of its component (at least one component) is eventually of constant sign on $[T, \infty) \subset [T_y, \infty)$.

In this paper we shall study oscillatory properties of solutions of differential systems (S_λ) with deviating arguments of mixed type, which are in general essentially different from those of ordinary $(h_i(t) \equiv t, i = 2, 3, \dots, n, g_m(t) \equiv t,$