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Some monomials in the universal Wu classes

Dedicated to Professor Shôrô Araki on his sixtieth birthday

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§1. Introduction

Let BO be the space which classifies stable real vector bundles. Then, its mod 2 cohomology $H^*(BO; Z_2)$ is the polynomial algebra over Z_2 on the (universal) Stiefel-Whitney classes $w_i \in H^i(BO; Z_2)$, $i \geqq 1$ (cf. [2], [6]). Moreover, the Steenrod squaring operation on $H^*(BO; Z_2)$ is given by

$$(1.1) \qquad Sq^j w_i = \sum_{t=0}^{j} \binom{i-1-t}{j-t} w_{i+j-t} w_t \qquad \text{for } 0 \leqq j < i\,,$$

where $\binom{a}{b}$ is the binomial coefficient and $w_0 = 1$ (cf. [7]).

Let $v_i \in H^i(BO; Z_2)$ be the (universal) Wu classes (cf. [1], [4], [5]) defined inductively by

$$(1.2) \qquad v_0 = w_0 = 1 \qquad \text{and} \qquad w_i = \sum_{k=0}^{i} Sq^k v_{i-k}\,, \quad i \geqq 1\,.$$

Then, the Wu class v_i is the polynomial

$$v_i = v_i(w_1, w_2, \ldots) \quad \text{with coefficients in } Z_2$$

on the Stiefel-Whitney classes w_j's, which can be described exactly by using (1.1-2) and the properties of the Steenrod operations, but it is not so easy in general to see the explicit form of this polynomial. In [8, Cor.], we find all monomials $w_{i_1} \ldots w_{i_s}$, $i_1 > \cdots > i_s \geqq 1$, which appear in $v_i(w_1, w_2, \ldots)$ with coefficient 1.

The purpose of this paper is to study the monomials of the form w_i^2 or $w_j w_1^k$, $j \geqq 2$, and to prove the following two theorems.

THEOREM 1.3. *In the polynomial $v_i(w_1, w_2, \ldots)$, the monomial w_j^2, $2j = i$, appears with coefficient* 1 *when and only when*

$$i = a \geqq 2\,, \quad or \quad i = a + b\,, \quad a > b \geqq 2\,,$$

where a and b are all powers of 2.