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Real hypersurfaces with harmonic Weyl tensor of a complex space form

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Abstract. We study real hypersurfaces of a complex space form $M_n(c)$. The purpose is to give another characterization of pseudo-Einstein hypersurfaces and then to prove that there are no real hypersurfaces with harmonic Weyl tensor of $M_n(c)$, $c \neq 0$, $n \geq 3$.

Introduction

A complex n-dimensional Kaehler manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. A complete and simply connected complex space form is a complex projective space P_nC, a complex Euclidean space C_n or a complex hyperbolic space H_nC, according as $c > 0$, $c = 0$ or $c < 0$. The induced almost contact metric structure of a real hypersurface of $M_n(c)$ is denoted by (ϕ, ξ, η, g).

Now, there exist many studies of real hypersurfaces of a complex space form. One of the first researches is the classification of homogeneous real hypersurfaces of a complex projective space P_nC by Takagi [12]. Some real hypersurfaces of a complex space form $M_n(c)$, $c \neq 0$, are characterized under the conditions for the shape operator (or principal curvatures) and one of the structure tensors. In particular, a real hypersurface M of $M_n(c)$, $c \neq 0$, is said to be *pseudo-Einstein* if the Ricci tensor S' satisfies

$$S' = ag + b\eta \otimes \eta ,$$

where a and b are some functions on M. The structure of pseudo-Einstein hypersurfaces is investigated by Cecil and Ryan [2], Kon [6] and Montiel [9].

On the other hand, some studies about the non-existence for real hypersurfaces under natural linear conditions which can be imposed on S' or $\nabla S'$ have been made by Kimura [5], Kon [6] and Montiel [9]. It is seen in [6] and [9] that there are no Einstein real hypersurfaces of $M_n(c)$, $c \neq 0$, $n \geq 3$. In particular, it is proved by Kim [4] and Kwon and one of the authors [7] that there are no real hypersurfaces with harmonic curvature of $M_n(c)$, $c \neq 0$, $n \geq 3$, on which the structure vector ξ is principal. Recently, the first author [3]

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