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Extensions of Poisson algebras by derivations

Dedicated to the memory of Professor Shigeaki Tôgô

Fujio KUBO and Fumitake MIMURA*

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Introduction

The alternating Schouten product was studied in a totally algebraic way in Bhaskara and Vismanath [3]. In this paper we shall be first concerned with this product and show that $[P, \hat{Q}] = 0$ if and only if $[P, Q] = 0$ and $(p-1)\,\mathrm{Alt}\,(P \otimes Q) = 0$ for alternating multiderivations P and Q of degree p and $q-1$ respectively, where $\hat{Q} = \mathrm{Alt}\,(q\bar{Q})$ is an alternating multilinear map of degree q (Theorem 2).

We shall then study an extension of a Poisson algebra by an derivation which is the abstract concept of a generalized Poisson algebra introduced by Berezin [2], while Kubo and Mimura [4] and Kubo [5] worked on abstract Poisson algebras, especially Poisson Lie structures on some polynomial algebras and their factor algebras. Let F be a Poisson algebra with bracket $[,]$ and D a derivation of the associative algebra F. We define a D-extension $(F, \langle , \rangle)$ of F whose bracket $\langle , \rangle$ on F is given by $\langle a, b\rangle = [a, b] + D(a)b - aD(b)$ for a, $b \in F$. By using Theorem 2 we give an equivalent condition to that an algebra $(F, \langle , \rangle)$ is a Lie algebra. Then we consider an extension of a Poisson algebra constructed from the three dimensional split simple Lie algebra.

Throughout this paper let $\mathfrak{k}$ be a field of characteristic zero and F a commutative associative algebra over $\mathfrak{k}$ with unit.

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Alternating Schouten products of multiderivations

Notations and terminology are based on Bhaskara and Viswanath [3]. For the sake of convenience we list the terms that we use here.

For $p \geqq 1$, we denote by $L_p(F)$ the set of all multilinear maps of F into itself of degree p. We define $L_0(F) = F$ and $L_{-1}(F) = 0$.

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