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Kiguradze classes for radial entire solutions of higher order quasilinear elliptic equations

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0. Introduction

We consider higher order quasilinear elliptic partial differential equations of the form

$$(0.1)\qquad (-1)^m \Delta^m u + \sigma f(|x|, u, \Delta u, \dots, \Delta^{m-1} u) = 0, \qquad x \in \boldsymbol{R}^N,$$

where $m \geqq 2$, $N \geqq 3$, $\sigma = +1$ or $\sigma = -1$, $|x|$ is the Euclidean length of $x = (x_1, \dots, x_N)$, and Δ^i denotes the i-th iterate of the N-dimensional Laplacian $\Delta = \sum_{k=1}^N \partial^2/\partial x_k^2$. It is always assumed that the function f in (0.1) is continuous on $[0, \infty) \times \boldsymbol{R}^m$ and satisfies the sign condition

$$(0.2)\qquad u_0 f(t, u_0, u_1, \dots, u_{m-1}) \geqq 0, \ \not\equiv 0 \qquad \text{for } (t, u_0, u_1, \dots, u_{m-1}) \in [0, \infty) \times \boldsymbol{R}^m.$$

A prototype of (0.1) satisfying (0.2) is the multi-dimensional generalized Emden-Fowler equation

$$(0.3)\qquad (-1)^m \Delta^m u + \sigma p(|x|)|u|^\gamma \operatorname{sgn} u = 0, \qquad x \in \boldsymbol{R}^N,$$

where $\gamma > 0$ and p is continuous on $[0, \infty)$ and $p(t) \geqq 0$, $\not\equiv 0$ for $t \geqq 0$.

We are concerned with the problem of existence (and nonexistence) of radial entire solutions of (0.1) which have no zero in $\boldsymbol{R}^N$. By a radial entire solution of (0.1) we mean a radially symmetric function $u(|x|) \in C^{2m}(\boldsymbol{R}^N)$ which satisfies (0.1) at every point of $\boldsymbol{R}^N$. The study of this problem was initiated by Walter [20, 21] and followed by Walter and Rhee [22], Kusano and Swanson [13], Kusano, Naito and Swanson [10–12], and Usami [19]. In particular, it is shown in [11] that the equation

$$(0.4)\qquad (-1)^m \Delta^m u + \sigma f(|x|, u) = 0, \qquad x \in \boldsymbol{R}^N,$$

generalizing (0.3), may possess a variety of positive or negative radial entire solutions with different types of asymptotic behavior as $|x| \to \infty$.

The purpose of this paper is to provide a theory which unifies and furthers basic theories developed in [10–13] and which enables us to obtain detailed information about the structure of radial entire solutions with no zero of equation (0.1). Our theory is based on the fact (Theorem 2.1 below) that a