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Error bounds for asymptotic expansions of some distributions in a multivariate two-stage procedure

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1. Introduction

In this paper we obtain error bounds for asymptotic expansions of the distributions of test statistics in a multivariate two-stage procedure. These statistics can be expressed as a scale mixture of a chi-square variable χ_p^2 with p degrees of freedom, i.e.

$$X = \sigma\chi_p^2 , \tag{1.1}$$

where σ is a positive-valued random variable and is independent of χ_p^2. The scale factor σ is defined by $\sigma = \frac{1}{p}\mathrm{tr}(S^{-1})$ for the case of a multivariate population, where nS is distributed as a Wishart distribution $\mathrm{W}_p(I_p, n)$. For the case of two multivariate populations, $\sigma = \frac{1}{2}(\sigma_1 + \sigma_2)$, where $\sigma_i = \frac{1}{p}\mathrm{tr}(S_i^{-1})$, $i = 1, 2$, and n_iS_i are independently distributed as $\mathrm{W}_p(I_p, n_i)$. Hyakutake and Siotani [6] obtained asymptotic expansions of the distribution of X in the two cases. Our purpose of this paper is to obtain explicit bounds for the approximations based on these asymptotic expansions. The method used is based on a general result (see, e.g., Fujikoshi and Shimizu [5]) for scale mixtures of the gamma distribution.

In the use of the result it is necessary that we evaluate the exact moments of $\sigma^{\pm 1}$. But it is generally difficult to obtain the exact ones in these cases. In this paper we derive appropriate approximations for their moments with error bounds. Using these results, we will obtain error bounds for asymptotic expansions of the distribution of X.

2. Scale mixture of a chi-square distribution

Here, we consider a scale mixture of a chi-square variable with p degrees of freedom, i.e.

$$X = \sigma\chi_p^2 . \tag{2.1}$$