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Inertial manifold theory for a class of reaction-diffusion equations on thin tubular domains

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§1. Introduction

During the past several decades, reaction-diffusion equations have been proposed to understand spatio-temporal structures of nonlinear phenomena arising in population ecology (Cantrell and Cosner [2], Kan-on and Yanagida [25], Levin [29], Mimura et al. [33]–[37]), neurobiology (Henry [19]), fission reactors (Leung [28]), chemical reactions (Fife [15], Smoller [42]), combustions (Ei et al. [11]–[13]) and other applied sciences. For the qualitative study of solutions of these equations, fruitful mathematical methods have been extensively developed in the field of applied analysis (Ei [9], Ei and Mimura [10], Mimura et al. [34], for instance).

Most of reaction-diffusion equations are described by the following semilinear parabolic system of equations:

$$u_t = div(D(x, u)\nabla u) + f(x, u), \qquad (t, x) \in (0, \infty) \times \Omega, \tag{1.1}$$

where Ω is a bounded domain in $\mathbf{R}^n$, $u = (u_1, \cdots, u_m) \in \mathbf{R}^m$, $D(x, u)$ is a nonnegative definite matrix and f is a kinetic function from $\mathbf{R}^n \times \mathbf{R}^m$ into $\mathbf{R}^m$. In most applications, D is a constant diagonal matrix and f is independent of x. The resulting system is simply

$$u_t = D\Delta u + f(u), \qquad (t, x) \in (0, \infty) \times \Omega. \tag{1.2}$$

The variables u usually denote the quantities such as densities of biological populations in ecology, concentrations of substances in chemical reaction, for instance. In qualitatively understanding the behavior of solutions to (1.1) or (1.2), the studies of existence and stability of equilibrium and periodic solutions to (1.1) and (1.2) are very important. In fact, it is known that stable spatially inhomogeneous equilibrium solutions play, among other things, an important role in the formation of patterns arising in reacting and diffusing medium.

As for stable spatially inhomogeneous equilibrium solutions, we have the following problem: How is the relation between the stability of spatially inhomogeneous equilibrium solutions and the shape of domain Ω. Along this