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# *On FC-solvable Groups*

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In this note, we shall show that some properties which hold true in finite groups and solvable groups satisfying the maximal condition for subgroups (say S-groups, after K. A. Hirsch) may be available for FC-solvable groups satisfying the maximal condition for subgroups, and that for FC-solvable groups the problem concerning the finiteness of finitely generated and torsion groups holds affirmative. The proofs are based on a method of K. A. Hirsch which is used in the case of S-groups [2; 3], that is, a method which reduces the problem concerned to the case of some finite factor groups.

A finite system of subgroups of a group $G$,

$$E=G_0\subset G_1\subset G_2\subset\cdots\quad G_k=G$$

beginning with the unit subgroup and ending in $G$ itself, is called an *FC-solvable series* of $G$ if every subgroup $G_i$ is a proper normal subgroup of $G_{i+1}$ and every factor group $G_{i+1}/G_i$ is an FC-group; $i=0, 1,\cdots,k-1$. An FC-solvable series in which every subgroup $G_i$ is a normal subgroup of $G$ is called an *FC-solvable normal series* of $G$. And a group $G$ is called *FC-solvable* if it has an FC-solvable series. After P. Hall, we denote simply the maximal condition for subgroups by Max.

Now, we proceed to prove the properties.

Proposition 1. *The FC-solvable groups satisfying Max are the finite extensions of torsion-free solvable groups.*

Proof. Let $G$ be an FC-solvable group satisfying Max. It is sufficient to prove only in the case where the group $G$ is infinite. From the result of A. M. Duguid and D. H. McLain ([1] Theorem 3), we can see that $G$ has an FC-solvable normal series. Let $G_0\subset G_1\subset\cdots\subset G_k$ be an FC-solvable normal series of $G$. Among the factor groups $G_i/G_{i-1}(i=1, 2,\cdots,k)$, at least one factor group is infinite. Let $l$ be the smallest of the suffix $i$ of infinite factor groups $G_i/G_{i-1}$. Then the factor group $G_l/G_{l-1}$ is an infinite FC-group and the group $G_{l-1}$ is finite. Therefore the group $G_l$ is an infinite FC-group. Moreover, $G_l$ is finitely generated, so its center has a finite index in it. Therefore, in the group $G_l$ there exists a torsion-free abelian subgroup $A$ which is normal in G. Let $N$ be a maximal normal subgroup of $G$ which is torsion-free and solvable and contains A. Then the factor group $G/N$ is finite. For, suppose that the group $G/N$ is infinite. The group $G/N$ is also FC-solvable group satisfying Max, so $G/N$ has a torsion-free, solvable and normal subgroup for the same reason as the above. Therefore $G$ has a torsion-free normal subgroup which is solvable