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# *Invariant Manifolds under a Certain Transformation*

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## 1. Introduction.

In this note, we are concerned with a manifold invariant under the real transformation $\Sigma$ of the form

$$z' = Cz + h(z) \tag{1}$$

where

$z$ and $z'$ are $n$-dimensional real vectors;

$C$ is a real $n \times n$-matrix such that some of its characteristic roots are less than unity in absolute value;

$h(z)$ is a real continuous function of $z$ such that $h(0)=0$ and, for any $\varepsilon > 0$, there exists a positive number $\delta = \delta(\varepsilon)$ such that

$$|h(z_1) - h(z_2)| \leqq \varepsilon |z_1 - z_2|^{1)}$$

whenever $|z_1|, |z_2| \leqq \delta$.

The present problem is connected closely with the conditional stability of the solutions of the differential equations, because the iteration of the transformation (1) converges on the invariant manifold in question.

The case where $h(z)$ is analytic was discussed already by M. Urabe [1]. In the present note, the same results will be proved for the more general case where $h(z)$ is a function described above.

## 2. The functional equation for an invariant manifold.

If we transform the variable $z$ to $\bar{z}$ by a real linear transformation $z = P\bar{z}$, the initial transformation (1) is written as

$$\bar{z}' = P^{-1}CP\bar{z} + P^{-1}h(P\bar{z}). \tag{2}$$

Clearly we can choose $P$ so that $P^{-1}CP$ may have the form

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1) $|\cdots|$ means the norm such that $|v| = \sum_i |v^i|$ for a vector $v = v^i$ and $|A| = \max_j \sum_i |a_{ij}|$ for a matrix $A = (a_{ij})$.