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Groupoid and Cohomology with Values in a Sheaf of Groupoids

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A. Grothendieck developed in [1]* systematically the notion of fibre space on a topological space X with structure sheaf $\mathfrak{G}$, where $\mathfrak{G}$ is any sheaf of groups, and the notion of 1-cohomology set $H^1(X, \mathfrak{G})$ of X with values in $\mathfrak{G}$. He showed the important relation between the elements of $H^1(X, \mathfrak{G})$ and the classes of fibre spaces on X with structure sheaf $\mathfrak{G}$. And he obtained the exact sequence of the cohomology sets (dim. 0 and 1) of X with values in sheaves of groups on X.

A. Haefliger introduced in [2] the cohomology sets $H^0(X, \mathfrak{P})$ and $H^1(X, \mathfrak{P})$ of X with values in a sheaf of groupoids $\mathfrak{P}$ on X. And under the assumption that $\mathfrak{P}$ is transitive, he proved that there exists a one-to-one correspondence between $H^1(X, \mathfrak{P})$ and $H^1(X, \mathfrak{G}^f)$, where $\mathfrak{G}^f$ is the sheaf of groups associated to an element f of $Z^1(\mathfrak{U}, \mathfrak{P})$.

In this paper, it is shown that we can obtain an exact sequence of the cohomology sets (dim. 0 and 1) of X with values in sheaves of groupoids on X. We deal with the inverse problem of the relation between $\mathfrak{P}$ and $\mathfrak{G}^f$ which was shown by A. Haefliger: When a sheaf of groups $\mathfrak{G}$ on X is given, we may introduce a sheaf of transitive groupoids $\mathfrak{P}$ on X and an element f of $Z^1(\mathfrak{U}, \mathfrak{P})$ such that we have a one-to-one correspondence $H^1(X, \mathfrak{P}) \rightarrow H^1(X, \mathfrak{G}^f)$ where the latter set can be identified with $H^1(X, \mathfrak{G})$. And when the sheaf of transitive groupoids $\mathfrak{P}$ has a unit section, it is shown that there exists a sheaf of groups $\mathfrak{G}$, which is simpler than $\mathfrak{G}^f$, such that $H^1(X, \mathfrak{P})$ corresponds one-one to $H^1(X, \mathfrak{G})$.

In the first half part, we prepare ourselves for treating the above-mentioned problems. In §1, we prove that two systems of axioms in the definition of groupoid—those in A. Haefliger's paper [2] and in P. Dedecker's [3]—are equivalent, and in §§2–3 we introduce the concept of normal subgroupoid and quotient groupoid. In §4, we refer to a representation of groupoid.

In the last half part, we solve the above-mentioned problems, applying the results in §§1–4 and introducing the concept of groupoid extension of a sheaf of groups.

§1. Axioms of groupoid.[2][3] A groupoid is a set Π which has a composition law $(x, y) \rightarrow xy$, defined for some pairs of elements x, y ($\in \Pi$) and

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* The numbers in brackets refer to References at the end of this paper.