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Recursive Games with Infinitely Many Strategies

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In our previous paper [1] in which certain stochastic games were shown to be strictly determined, we have first defined a dummy game which may be considered a linearlization of stochastic games, and then by using the principle of contraction we have shown the existence of the principal value vector thereof which turned out to be the value vector of the stochastic games. This paper is a continuation of the one [1] just cited, and by the same method as above we shall show that some recursive games (a recursive game is originated by *H.* Everett [2]) are strictly determined.

Throughout this paper we use the notations and some of the results which were obtained in [1].

First we begin with the definition of recursive games. Suppose we are given N positions 1, 2, $\cdots$, N. To each position k we consider a game

$$\Gamma_k = (A_k, B_k, g_k, \mathfrak{M}_k, \mathfrak{N}_k)$$

called a component game of the recursive game, which will be defined below. Let Players 1 and 2 choose a pair $(a, b) \in A_k \times B_k$. Then the transition probabilities $p_{kl}(a, b)$ and the stop probabilities $p_{k0}(a, b)$ are given. Here $p_{kl}(a, b)$ denotes the probability with which the game Γ_k moves to the next one Γ_l and $p_{k0}(a, b)$ denotes the probability with which the game stops at this position k. Let $\mathfrak{A}_k$ (resp. $\mathfrak{B}_k$) be a σ-algebra of subsets of A_k (resp. B_k) such that $\mathfrak{A}_k$ (resp. $\mathfrak{B}_k$) contains one point set (a) for any $a \in A$ (resp. (b) for any $b \in B$), and $\mathfrak{C}_k$ be the smallest σ-algebra of subsets of $A_k \times B_k$ which contains the Cartesian product $\mathfrak{A}_k \times \mathfrak{B}_k$. We assume that $p_{kl}(a, b)$, $p_{k0}(a, b)$ are bounded and $\mathfrak{C}_k$-measurable. The recursive game $\boldsymbol{\Gamma}$ is defined as the collection of all Γ_k, p_{kl}, p_{k0}, $k, l = 1, 2, \cdots, N$, where we assume

(i) the pay-offs $g_k(a, b)$ are bounded and $\mathfrak{C}_k$-measurable over $A_k \times B_k$ for every k,

(ii) payments can take place when and only when the game stops,

(iii) strategy spaces (A_k, δ^{g_k}), $(A_k, \delta^{p_{kl}})$ are precompact for every $k, l = 1, 2, \cdots, N$,

and

(iv) the transition probabilities $p_{kl}(a, b)$ are $\mathfrak{C}_k$-measurable over $A_k \times B_k$ for every k, l, and the stop probabilities $p_{k0}(a, b)$ are non-negative for every k.

Now we shall define dummy games associated with $\boldsymbol{\Gamma}$. Suppose Player 1