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Analysis of Partially Balanced Incomplete Block Designs

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1. Introduction and Summary.

In 1959, J. Ogawa [7] dealt with the analysis of association algebra, introduced by R. C. Bose [2], and the relationship algebra, introduced by A. T. James [5], of the partially balanced incomplete block designs (PBIBD). He obtained all of the irreducible constituents in the direct decomposition of the relationship algebra of the design. His results, however, were concerned with a restrictive case which might be considered to be regular PBIBD as was defined by Bose [1] in the case of group divisible incomplete block designs, because a restrictive condition "$BT_i^* = T_i^* B$ for all T_i^*" was imposed on the relationship matrices. Theorem I of this paper will throw light on the meaning of this restriction. Another restriction imposed implicitly on the relationship algebra of PBIBD, was that the ideal of the algebra related to the block relationship did not degenerate to zero.

We have succeeded in removing those restrictions and completing the analysis of partially balanced incomplete block designs. In PBIBD, NN' belongs to the association algebra $\mathfrak{A}$ and it will be seen in Theorem I of this paper that the magnitude of each density ρ of its spectral expansion in $\mathfrak{A}$ determines the property of the corresponding component of the treatment sum of squares (S. S.) to be either orthogonal to, or confounded with, or partially confounded with, the block space. The dimension of the relationship algebra $\mathfrak{R}$ and the unique decomposition of its unit element into mutually orthogonal principal idempotents will be given in Theorem II. Examination of the ideal related to block relationship will give an inequality which appears in Theorem III. Although our inequality is essentially the same with the one due to W. S. Connor and W. H. Clatworthy [3] in a certain sense, the former is more substantial than the latter in that it will make clear the significance of the reduction in the lower bound of the number of blocks in a PBIBD. It includes, as its special cases, those inequalities given by Fisher in balanced incomplete block designs (BIBD) and by Bose [1] in group divisible incomplete block designs. Complete Table for the analysis of variance of PBIBD will also be given.

2. Association scheme and association algebra.

We say, following Bose [2], that an association scheme is defined in a set