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On Limits of BLD Functions along Curves

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In the preceding paper [4], F-Y. Maeda proved that almost every Green line converges to one point on the boundary obtained by a certain compactification of a Green space, notably for the Kuramochi boundary. We shall use the contents of [4] freely. In this note we shall prove that every curve on a space $\mathscr{E}$ has a similar property, except for those belonging to a family with infinite extremal length.

Consider a space $\mathscr{E}$ in the sense of Brelot and Choquet [1]; $\mathscr{E}$ may not be a Green space. We begin with the definition of extremal length of a family Γ of locally rectifiable non-degenerate curves on $\mathscr{E}$. Any measurable function $\rho \geqq 0$ on $\mathscr{E}$ with the property that $\int_c \rho ds$ is defined and $\geqq 1$ for each $c \in \Gamma$ is called *admissible* (in association with Γ) and the *module* $M(\Gamma)$ of Γ is defined by $\inf_\rho \int \rho^2 dv$, where ρ is admissible and dv is the volume element. The *extremal length* of Γ is defined by $1/M(\Gamma)$. We shall say that *almost every* curve on $\mathscr{E}$ has a certain property if the module of the exceptional family vanishes. The definitions of an admissible ρ and the module need obvious modifications in case the dimension of $\mathscr{E}$ is two. However, we shall use higher dimensional phrases in the sequel.

Let $\bar{\mathscr{E}}$ be a topological space containing $\mathscr{E}$ such that $\mathscr{E}$ is everywhere dense in $\bar{\mathscr{E}}$ and any two points of $\bar{\mathscr{E}}$ are separated by a continuous function on $\bar{\mathscr{E}}$; $\bar{\mathscr{E}}$ may not be compact. We set $\Delta = \bar{\mathscr{E}} - \mathscr{E}$ and denote by $C_{\mathscr{E}}(\bar{\mathscr{E}})$ the family of functions consisting of the restrictions to $\mathscr{E}$ of all the bounded continuous functions on $\bar{\mathscr{E}}$.

A family $\mathscr{Q}$ of real functions on $\mathscr{E}$ is said to separate points of $\bar{\mathscr{E}}$ (Δ resp.) if, for any different $P_1, P_2 \in \bar{\mathscr{E}}$ (Δ resp.), there is $f \in \mathscr{Q}$ such that

$$\varliminf_{\substack{P \to P_1 \\ P \in \mathscr{E}}} f(P) > \varlimsup_{\substack{P \to P_2 \\ P \in \mathscr{E}}} f(P).$$

We shall say that a function has a limit (a finite limit resp.) along an open curve on $\mathscr{E}$ if it has a limit (a finite limit resp.) as the point moves on the curve in each direction.

Using the well-known inequality $M(\bigcup_n \Gamma_n) \leqq \sum_n M(\Gamma_n)$, we can prove the following theorem in a fashion similar to the proof of Theorem 1 of F-Y.