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Point-free Parallelism in Wilcox Lattices

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1. Introduction.

In the previous papers [4] and [5], I have investigated the properties of affine matroid lattices, using the parallelism given in [1], and I have seen that the points have significant roles. Hence this parallelism can not be applied to the non atomic lattices. Hsu [2] gave an apparently point-free parallelism, but in [4] Theorem (2.3), I have shown that this parallelism is coincident with that of [1].

In the present paper, I give a point-free parallelism using the modular elements instead of points, and applying to the Wilcox lattices, I obtain the same theorems as in [4] and [5].

In appendix, I investigate the modular centers of affine matroid lattices from the standpoint of the Wilcox lattice, and I obtain the same results as in the preceding paper [4].

2. Point-free parallelism in weakly modular symmetric lattices.

Definition (2.1). In a lattice L, we write $(a, b)M$ if $(c\cup a)\cap b=c\cup(a\cap b)$ for every $c\leqq b$. When b covers a, we write $a\lessdot b$.

In a lattice L with 0, $a\perp b$ means $a\cap b=0$, $(a, b)M$; and $a\parallel b$ means $a\cap b=0$, $(a, b)\bar{M}$ ($\bar{M}$ being the negation of the relation M). If $a\perp b$ implies $b\perp a$, then L is called a *symmetric lattice* (cf. [8] p. 495); and if $a\cap b\neq 0$ implies $(a, b)M$, then L is called a *weakly modular lattice* (cf. [4] (1.1)).

A relatively atomic, upper continuous, symmetric lattice is called a *matroid lattice* (cf. [5] (2.1)).

In this paper, we deal with a given lattice L with 0.

Definition (2.2). In a lattice L, a is called a *modular element* of L, if $(b, a)M$ for every $b\in M$ (cf. [6] p. 326). A point p, if it exists, is a modular element.

Remark (2.3). Especially when L is a weakly modular symmetric lattice, since $a\cap b\neq 0$ implies $(a, b)M$, a is a modular element if and only if $a\cap b=0$ implies $a\perp b$ for every $b\in L$.

Reference (2.4). In [1] p. 273, the parallelism in a matroid lattice L is