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Approximate Computation of Errors in Numerical Integration of Ordinary Differential Equations by One-step Methods

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1. Introduction

Given a differential equation

$$(1.1) \qquad y' = f(x, y),$$

where $f(x, y)$ is assumed to be a sufficiently smooth function.

In numerous papers [1−16][1], various methods are obtained for bounding or approximating the errors in numerical integration of (1.1) by one-step methods with the aids of the functions that bound or approximate the function $f_y(x, y)$, the truncation error and so on.

To avoid the use of such functions for practical purposes, in this paper, n steps of integration with a fixed step-size are considered as one step and a simple method is obtained for approximating the errors without computing explicitly any function other than $f(x, y)$. The method is illustrated by two numerical examples.

Since usually the step-size is not changed so often and the estimate of the error is not always necessary for each step of integration, it will not be a serious restriction to fix the step-size for the n steps of integration, and this method may be used as an integration method with a check on the accuracy of the numerical solution.

2. Preliminaries

In this paragraph and the next, we state five lemmas and one theorem without proof, but they are proved in paragraph 6.

In the sequel, for simplicity, we assume that $f(x, y)$ is defined, continuous and bounded in the strip

$$S : x_0 \leqq x \leqq x_0 + nh_0, \quad |y| < \infty,$$

and that the partial derivatives of $f(x, y)$ up to the necessary order, say s,

1) Numbers in square brackets refer to the references listed at the end of this paper.