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On Loop Extensions of Groups and M-cohomology Groups. II

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Introduction

In the previous paper [5][1], we discussed the problem of BM-extensions of a group by a group, that is, for given two groups G and Γ, the problem to determine all Bol-Moufang loop L's with the following properties[2]: (i) L has a normal subgroup G' which is isomorphic to G, (ii) $L/G' \cong \Gamma$, (iii) G' is contained in the nucleus of L. When we consider the case where L is a Bol-Moufang loop, it seems natural to consider the case where Γ is also a Bol-Moufang loop. In this paper we shall investigate the classification of all *BM*-extensions of a group G by a Bol-Moufang loop Γ. In this case, we shall modify the *M*-cohomology groups defined in the previous paper and classify all *BM*-extensions, using this new cohomology groups.

§1 will be devoted to the construction of the *M*-cohomology groups of a Bol-Moufang loop Γ over an abelian group G, and in §2, we shall first obtain the necessary and sufficient conditions for the existence of the *BM*-extension L of a group G by a Bol-Moufang loop Γ by making use of a *M*-factor set and a system of automorphisms of G, and next, using this result and the new *M*-cohomology groups we shall classify the set of all *BM*-extensions. The methods used in this paper are the same as those of the previous, and the results obtaind in this paper are as follows:

(i) *For a given group G with the center C, a Bol-Moufang loop Γ and a homomorphism $\theta: \Gamma \to Aut\, G/In\, G$*[3]*, the BM-extension of G by Γ exists if and only if an element of $H^{*3}(\Gamma, C)$ determined by G, Γ and θ is zero (Theorem 2). Especially in the case G is abelian, this element is always zero.*

(ii) *If the BM-extension exists for assigned G, Γ and θ, all non-equivalent BM-extensions are in one-to-one correspondence with the elements of the second M-cohomology group $H^{*2}(\Gamma, C)$ (Theorem 3, 4).*

§ 1. M-cohomology groups of a Bol-Moufang loop over an abelian group

In this section we shall extend the previous *M*-cohomology group of a

1) The number in the bracket referes to the references at the end of this paper.

2) A loop which satisfies the condition $a[b(ac)] = [a(ba)]c$ is called a Bol-Moufang loop.

3) Aut G means the group of all automorphisms of G and In G is the group of all inner automorphisms of G.