J. Sci. Hiroshima Univ. Ser. A-I
29 (1965), 253-270

A Remark on a Certain G-structure

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(Received September 25, 1965)

Introduction

Since the notion of G-structures on a differentiable manifold M was introduced by S. S. Chern [2][1] in 1953, a number of papers on this subject have been published by many writers, such as D. Bernard, R. S. Clark and M. Bruckheimer. Many structures which appear in differential geometry are closely related to the G-structures defined by certain special tensor fields whose components relative to some covering of M by moving frames are constants. Especially, among the G-structures defined by special vector 1-forms one finds the almost product, the almost complex and the almost tangent structures, etc..

As is well known, for such a G-structure we can define two tensors, that is, the Chern invariant and the Nijenhuis tensor. These two tensors play an impotant role in the theory of connections and the integrability of the G-structures. So far, however, we have known of the relation between them only in some special cases. For example, the Chern invariant vanishes if and only if the Nijenhuis tensor vanishes for almost product, almost complex and almost tangent structures [3]. The main purpose of this paper is to investigate how such a relation will be generalized in the case of the real G-structure defined by any special vector 1-form whose eigenvalues are all real.

As usual, we assume that all the objects we encounter in this paper are of class C^∞.

§ 1. Preliminaries.

In this section we introduce some general notions and symbols that will be used later on, and then state a main theorem.

1) *Chern invariant.* Let us assume that an m-dimensional differentiable manifold M has a G-structure, that is, the frame bundle over M admits a subbundle H with structure group G. Suppose Σ to be the torsion tensor of some structure connection, then the components $t\Sigma$ relative to any adapted frame are defined on H and P-valued, where $P=R^m\otimes R_m\wedge R_m$. Let e_i be the vectors of the natural basis for R^m, then $e_i^{jk}=e_i\otimes e^j\wedge e^k$ is a basis for P. We denote

(1) The number in bracket refers to the references at the end of the paper.