J. Sci. Hiroshima Univ. Ser. A-I
30 (1966), 91-107

On a One-step Method of Order 4

Hisayoshi SHINTANI

(Received March 15, 1966)

1. Introduction

Given a differential equation

$$y' = f(x, y) \tag{1.1}$$

and the initial condition

$$y(x_0) = y_0, \tag{1.2}$$

where $f(x, y)$ is assumed to be a sufficiently smooth function. We are concerned with the case where the equation (1.1) is integrated numerically by a one-step method of order 4.

It is well known that the one-step method of order 4 requires at least four evaluations of the derivative per step and that, in the course of numerical integration, if the same step-size is used twice in succession, the approximate value of the truncation error is obtained by integrating again with the double step-size. This method of approximating the truncation error requires at least three additional evaluations of the derivative per two steps of integration. Hence we raise the question whether there is or not a one-step formula of order 4 such that only one additional evaluation of the derivative makes it possible to approximate the truncation error for two steps of integration with the same step-size.

In this paper, it is shown that such a formula really exists and the method is illustrated by numerical examples.

2. Preliminaries

Put

$$x_j = x_0 + jh \qquad (j=1, 2, \cdots) \tag{2.1}$$

$$z_1 = y_0 + h\sum_{i=1}^{4} p_i k_i, \tag{2.2}$$

and