J. Sci. Hiroshima Univ. Ser. A-I
30 (1966), 75-90

Direct Methods for the Numerical Solution of Partial Difference Equations for a Rectangle

Hisayoshi Shintani

(Received March 15, 1966)

1. Introduction

The problem of solving approximately elliptic partial differential equations over a rectangle with Dirichlet boundary conditions is often reduced to the problem of solving the system of linear equations of the following form

$$(1.1)\qquad \begin{cases} A_1\boldsymbol{x}_1 - B_1\boldsymbol{x}_2 = \boldsymbol{\Gamma}_1, \\ -C_k\boldsymbol{x}_{k-1} + A_k\boldsymbol{x}_k - B_k\boldsymbol{x}_{k+1} = \boldsymbol{\Gamma}_k \qquad (k=2, 3, \cdots, m-1) \\ -C_m\boldsymbol{x}_{m-1} + A_m\boldsymbol{x}_m = \boldsymbol{\Gamma}_m, \end{cases}$$

where $\boldsymbol{x}_i$ and $\boldsymbol{\Gamma}_i$ $(i=1, 2, \cdots, m)$ are n-vectors and A_i, B_i and C_i are $n\times n$ diagonal or tridiagonal matrices [4][1].

The system (1.1) is usually solved by the iterative methods and the direct methods are rarely used because of the storage capacity [4]. Among the direct methods, however, there are the square root method [4], the hypermatrix method [2, 3, 13], the tensor product method [8, 9], and so on [1, 6, 7, 12, 14]. As G.E. Forsythe and W.R. Wasow [4] indicate, direct methods are of practical use when they need not so large storage space and the inverse matrices can be generated or the problem is reduced to the inversion of the matrices of the small order.

In this paper, direct methods are derived in an elementary manner for (1.1), for the periodic boundary problems of Poisson's equations and of one-dimensional heat equations, and for biharmonic equations. Stability of the numerical process is discussed in some cases.

2. Dirichlet problem

In this paragraph, we assume that the system (1.1) has a unique solution and that B_k $(k=1, 2, \cdots, m-1)$ are non-singular.

1) Numbers in square brackets refer to the references listed at the end of this paper.