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On a Capacitability Problem Raised in Connection with Linear Programming

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Introduction

Let X and Y be compact Hausdorff spaces, $\Phi(x, y)$ be a universally measurable[1] function on $X\times Y$ which is bounded from below, $g(x)$ be any function on X and $f(y)$ be a universally measurable function which is bounded from above. We denote by $\mathscr{M}$ the set of all non-negative Radon measures satisfying the inequality

$$\int\Phi(x, y)d\mu(y)\leqq g(x) \quad \text{on} \quad X .$$

In the case that $\mathscr{M}$ is not empty, the quantity

$$M=\sup\left\{\int fd\mu ;\ \mu\in\mathscr{M}\right\}$$

was considered by Ohtsuka [4] in connection with a generalization of a theorem in the theory of linear programming. In this paper, we consider the family $\mathscr{M}_K$ of measures in $\mathscr{M}$ supported by a compact subset K of Y and consider a similar quantity

$$M(K)=\sup\left\{\int fd\mu ;\ \mu\in\mathscr{M}_K\right\}$$

in the case that $\mathscr{M}_K$ is not empty. This quantity has a potential theoretic meaning. In fact, Fuglede [2] considered it in case $\Phi\geqq 0$, $g=1$ and $f=1$ and denoted it by cap K. We shall call it Fuglede's capacity in § 11.

For any set $A\subset Y$, we define in § 1 an inner quantity $M_i(A)$ and an outer quantity $M_e(A)$ from $M(K)$ in the same way as the inner capacity cap_*A and the outer capacity cap^*A were defined from cap K in [2]. Ohtsuka orally raised the question as to when $M_i(A)$ is equal to $M_e(A)$. We shall give an answer to this question in the present paper.

Kishi [3] examined this problem in the case that $X=Y$, $\Phi(x, y)=\Phi(y, x)>0$ for all x, $y\in X$, Φ is lower semicontinuous and $g=f=1$. His main result

1) A function on a compact set is universally measurable if it is measurable with respect to all Radon measures.