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Generalized Capacity and Duality Theorem in Linear Programming

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Introduction

Recently certain results in the theory of games and linear programming have been applied to potential theory. We mention M. Nakai [2], B. Fuglede [1] and M. Ohtsuka [4]. Our paper is along this line.

More precisely, the minimax theorem in the theory of games was applied to the theory of capacity in [4]. For a compact Hausdorff space K and an extended real-valued lower semicontinuous function Φ on $K\times K$ which is bounded below, the author established

$$\inf_{\mu\in\mathscr{U}}\ \sup_{x\in S_\mu}\int\Phi(x,\ y)d\mu(y)=\inf_{\nu\in\mathscr{U}}\ \sup_{y\in S_\nu}\int\Phi(x,\ y)d\nu(x) \tag{1}$$

and

$$\sup_{\mu\in\mathscr{U}}\ \inf_{x\in S_\mu}\int\Phi(x,\ y)d\mu(y)=\sup_{\nu\in\mathscr{U}}\ \inf_{y\in S_\nu}\int\Phi(x,\ y)d\nu(x)\ , \tag{2}$$

where $\mathscr{U}$ is the class of unit measures[1)] in K. See [3] for a simple proof of (1) in the case where K is discrete. We extend these results in the present paper. In §1 we consider a lower semicontinuous kernel, and generalize (1) by making use of a duality theorem in linear programming obtained in [5]. Next we are concerned with an upper semicontinuous kernel. A generalization of (2) is obtained there.

Let Φ be a function (called kernel) on $K\times K$ which is bounded above or below, and let g and f be upper or lower semicontinuous functions on K which are bounded above or below. We denote by $\mathscr{N}(\mathscr{N}^+$ resp.) the class of measures (non-zero measures resp.) μ satisfying $\int\Phi(x,\ y)d\mu(y)\leqq g(x)$ on S_μ, and by $\check{\mathscr{N}}(\check{\mathscr{N}}^+$ resp.) the class of measures (non-zero measures resp.) ν satisfying $\int\Phi(x,\ y)d\nu(x)\leqq f(y)$ on S_ν. We set

$$N=\sup_{\mu\in\mathscr{N}}\int fd\mu,\ N^+=\sup_{\mu\in\mathscr{N}^+}\int fd\mu,\ \check{N}=\sup_{\nu\in\check{\mathscr{N}}}\int gd\nu,\ \check{N}^+=\sup_{\nu\in\check{\mathscr{N}}^+}\int gd\nu$$

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1) Here and throughout our paper a measure means a non-negative Radon measure.