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Some Examples Related to Duality Theorem in Linear Programming

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The duality problems in linear programming may read as follows. Suppose an $m \times n$ matrix $A=(a_{ij})$, a column vector $\boldsymbol{b}=(b_1, \cdots, b_m)$ and a row vector $\boldsymbol{c}=(c_1, \cdots, c_n)$ are given.

The primal problem: *Find a column vector* $\boldsymbol{u}=(u_1, \cdots, u_n)$ *which maximizes the linear form* $\boldsymbol{cu}$ *subject to the conditions* $A\boldsymbol{u} \leqq \boldsymbol{b}$ *and* $\boldsymbol{u} \geqq 0$.

The dual problem: *Find a row vector* $\boldsymbol{v}=(v_1, \cdots, v_n)$ *which minimizes the linear form* $\boldsymbol{vb}$ *subject to the conditions* $\boldsymbol{v}A \geqq \boldsymbol{c}$ *and* $\boldsymbol{v} \geqq 0$.

In each problem a vector satisfying the required conditions is called feasible, and if it attains the maximum or minimum it is called optimal.

These problems can be represented by the following tableau:

$(\geqq 0)$	u_1	$\cdots$	u_j	$\cdots$	u_n	$\leqq$
v_1	a_{11}	$\cdots$	a_{1j}	$\cdots$	a_{in}	b_1
.	.	$\cdots$	.	$\cdots$	.	.
v_i	a_{i1}	$\cdots$	a_{ij}	$\cdots$	a_{in}	b_i
.	.	$\cdots$	.	$\cdots$	.	.
v_m	a_{m1}	$\cdots$	a_{mj}	$\cdots$	a_{mn}	b_m
$\geqq$	c_1	$\cdots$	c_j	$\cdots$	c_n	min / max

By taking inner products of the row of u's with the rows of A and the row of c's, we obtain the constraints $A\boldsymbol{u} \leqq \boldsymbol{b}$ and the linear form $\boldsymbol{cu}$ of the primal; the inner products of the column of v's with the columns of A and the column of b's yield the dual constraints $A\boldsymbol{v} \geqq \boldsymbol{c}$ and the linear form $\boldsymbol{vb}$.

Associated with these problems is the following well-known theorem:

The Duality Theorem. If the primal is feasible and if $\sup \boldsymbol{cu} < \infty$, *then there exist optimal solutions in the dual as well as in the primal, and moreover the extremal values of the linear forms coincide, i.e.,* $\max \boldsymbol{cu} = \min \boldsymbol{vb}$.

In the foregoing paper [1], M. Ohtsuka investigated the problems in a very general situation, and obtained extensions of the duality theorem. We refer necessary notions and notations to [1]. We shall show in the present paper that the conditions imposed in Ohtsuka's Theorems 2 and 3 are in a way necessary. Actually, even if $\mathscr{M} \neq \emptyset$, $-\infty < M < \infty$ and Φ, f and g are