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On a Trace Theorem for the Space $H^{\mu}(R^N)$

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Consider the space $H_m(R^N)$, R^N being an N-dimensional Enclidean space, composed of temperate distributions u defined in R^N such that the Fourier transform $\hat{u}(\xi)$ is a locally integrable function satisfying

$$\int_{\Xi^N} |\hat{u}(\xi)|^2(1+|\xi|^2)^m d\xi < \infty .$$

Let m be a positive number $> \frac{1}{2}$ and l the largest integer such that $l < m - \frac{1}{2}$. It is known that the trace mapping

$$u \in H_m(R^N) \to \left(u(x',0), \cdots, \frac{\partial^l}{\partial x_N^l} u(x',0)\right) \in \prod_{j=0}^{l} H_{m-j-\frac{1}{2}}(R^{N-1})$$

is an epimorphism, where x' stands for $(x_1, x_2, \cdots, x_{N-1})$.

$H_m(R^N)$ is a particular instance of the spaces $H^{\mu}(R^N)$, μ being a temperate weight function defined in Ξ^N. The discussion on the spaces $H^{\mu}(R^N)$ is given in full detail in L. Hörmander [1] and in L.R. Volevič and B.P. Paneyah [5]. As a result of J. L. Lions' theorems on the Hilbert spaces [2], the trace theorem as mentioned above remains valid for $H^{\mu}(R^N)$ when $\mu(\xi)$ is equivalent to

$$\mu_1(\xi') + |\xi_N|^a \mu_2(\xi')$$

where $\mu_1(\xi')$, $\mu_2(\xi')$ are temperate weight functions in Ξ^{N-1}.

Recently M. Pagni has shown the theorem for a special $H^{\mu}(R^N)$, to which Lions' theorem is not applicable [3].

Our main aim of this paper is to investigate the trace theorem of the above type for general $H^{\mu}(R^N)$. We have obtained the necessary and sufficient conditions for the validity of the theorem (cf. Theorem 1 below). It is to be noticed that a sufficient condition to the effect that $\mu(\xi', 2\xi_N) \geqq C\mu(\xi', \xi_N)$, C being a constant, seems convenient to guarantee the theorem in most cases as enumerated above.

1. Notations and Terminologies. Let R^N be an N-dimensional Euclidean space and let Ξ^N be its dual space. For $x = (x_1, \cdots, x_N) \in R^N$ and $\xi = (\xi_1, \cdots, \xi_N) \in \Xi^N$, the scalar product $\langle x, \xi \rangle$ and the length of the vector