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Two-step Processes by One-step Methods of Order 3 and of Order 4

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1. Introduction

Given a differential equation

$$y' = f(x, y) \tag{1.1}$$

and the initial condition $y(x_0) = y_0$, where $f(x, y)$ is assumed to be a sufficiently smooth function. We are concerned with the case where the equation (1.1) is integrated numerically by one-step methods of order 3 and of order 4.

It is well known that the one-step methods of order 3 such as Kutta method and those of order 4 such as Runge-Kutta method require three and four evaluations of the derivative respectively. It is also known that, if the same step-size is used twice in succession, an approximate value of the truncation error can be obtained by integrating again with the double step-size. This method of approximating the truncation error requires, per two steps of integration, eight and eleven evaluations of $f(x, y)$ for any one-step method of order 3 and for that of order 4 respectively.

In our previous paper [19][1], it has been shown that there exists a one-step formula of order 4 such that, after two steps of integration with the same step-size, only one additional evaluation of the derivative makes it possible to approximate the truncation error. In that formula, however, four values of $f(x, y)$ evaluated in the first step of integration are not used explicitly in the second step. Thus there remains a possibility of reducing the number of evaluations of the derivative by utilizing all the values of the derivative computed already.

In this paper, it is shown that there exist one-step integration formulas of order 3 and those of of order 4 such that approximate values z_1 and z_2 of $y(x_0+h)$ and $y(x_0+2h)$ and an approximation to their truncation errors can be obtained with five and seven evaluations of $f(x, y)$ respectively. Finally two numerical examples are presented.

1) Numbers in square brackets refer to the references listed at the end of this paper.