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On Finite Geometries and Cyclically Generated Incomplete Block Designs

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1. Introduction

C. R. Rao [3], [4] generalized certain theorems known as the difference theorems of R. C. Bose [1] and derived a method of constructing difference sets which cyclically generate balanced incomplete block (BIB) designs. The main results were derived with the help of a compact representation of d dimensional linear subspaces (flats) in a $t(>d)$ dimensional finite projective space and also in Euclidean space. The notion of the cycle of a flat was introduced there in order to investigate the structure of the family of flats and the following general propositions were conjectured:

Proposition 1 (Rao) *In* PG(t, m), *if* $r_1, r_2, \cdots, r_p$ *are integers such that*

(a) $0<r_1<r_2<\cdots<r_p<t$,

(b) $(m^{d+1}-1)/(m^{r_i+1}-1)=s_i$ *integral for all* i,

(c) $(d+1)/(r_i+1)=t_i$ *integral for all* i,

(d) $(r_{i+1}+1)/(r_i+1)=l_i$ *integral for all* i,

(e) $(m^{t+1}-1)/(m^{r_i+1}-1)=\theta_i$ *integral for all* i,

then there are

$$y_i=(n_i-n_{i+1})/\theta_i \quad \textit{where} \quad n_i=\binom{\theta_i}{t_i}\Big/\binom{s_i}{t_i}$$

initial flats of cycle θ_i $(i=1, 2, \cdots, p)$ *and*

$$\eta=(b-n_1)/v$$

initial flats of cycle v *from which the totality of the* d*-flats can be generated.*

Proposition 2 (Rao) *In* EG(t, m), *if* $h=p_0p_1^{i_1}p_2^{i_2}\cdots$ ($p_0=1$ *and* p*'s are primes such that* $p_i<p_{i+1}$) *is the highest common factor* (H.C.F.) *of* d *and* t, *then the* d*-flats passing through the origin* (0) *will have cycles of the form* $\theta_{js}=(m^t-1)/(m^{r_{js}}-1)$ *where*

$$r_{js}=p_1^{i_1}p_2^{i_2}\cdots p_j^s \qquad (j=0, 1, \cdots;\ s=0, 1, \cdots, i_j).$$