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Order of the Identity Class of a Loop Space

Masahiro Sugawara

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Introduction.

For a topological space X with base point, its loop space ΩX is a homotopy-associative H-space with a homotopy-inverse, and so the set of homotopy classes of continuous maps from a topological space Y into ΩX, fixing base point, forms a group $\pi_0(Y; \Omega X)$. Consider the class

$$\iota_{\Omega X}=[1_{\Omega X}]\in\pi_0(\Omega X; \Omega X)$$

of the identity map $1_{\Omega X}$ of ΩX onto itself, and call the order of $\iota_{\Omega X}$ simply the *loop-order* of X.

The loop-order is clearly a homotopy type invariant. In this note, we discuss its general properties, where the dual situation to the suspension-order of Toda [2] may be seen.

1. Preliminary and definition.

For each topological space X, we always associate a point $*$, called the base point. (Continuous) maps and homotopies considered are base point preserving.

The set of the homotopy classes of maps $f:(X, *)\rightarrow(Y, *)$ is denoted by

$$\pi_0(X; Y).$$

Let $\alpha\in\pi_0(X; Y)$ and $\beta\in\pi_0(Y; Z)$ be the classes of maps $f: X\rightarrow Y$ and $g: Y\rightarrow Z$ respectively, then the composition $\beta\circ\alpha\in\pi_0(X; Z)$ is the class of the composition $g\circ f$ of maps. The formula $\beta\circ\alpha=f^*(\beta)=g_*(\alpha)$ defines two mappings

$$f^*: \pi_0(Y; Z)\rightarrow\pi_0(X; Z) \quad\text{and}\quad g_*: \pi_0(X; Y)\rightarrow\pi_0(X; Z).$$

The *loop space* ΩX of X is the space of all loops $w: (I, \dot{I})\rightarrow(X, *)$ ($I=[0, 1]$, $\dot{I}=\{0, 1\}$) with compact-open topology, and the constant loop is its base point. In this note, we assume that spaces are simply connected whenever their loop spaces are considered, and so ΩX is arcwise connected.

The product $\mu: \Omega X\times\Omega X\rightarrow\Omega X$ of the H-space ΩX is defined by

$$(w_1, w_2)(t)=w_1(2t)\quad(0\leqq t\leqq 1/2), \quad =w_2(2t-1)\quad(1/2\leqq t\leqq 1),$$