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An Example of Non-minimal Kuramochi Boundary Points

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Introduction.

Z. Kuramochi [3] constructed an example of a plane domain whose Kuramochi boundary contains non-minimal points. However, he showed only the existence of non-minimal points and did not determine the distribution of such points. In this note, applying his idea, we shall give an example of a domain in the d-dimensional Euclidean space R^d ($d \geq 2$) whose Kuramochi boundary contains non-minimal points and for which we are able to determine the distribution of non-minimal points completely. Our example is similar to, but simpler than Kuramochi's.

More precisely, let F be a compact set in R^d such that components of F cluster to the origin and F lies on the hyperplane $P = \{x = (x_1, \cdots, x_d); x_d = 0\}$. Unber certain conditions on F, we shall see that the Kuramochi boundary of $R^d - F$ corresponding to the origin is homeomorphic to the closed interval $[-1, 1]$, the points corresponding to 1 and -1 are minimal and the other points are non-minimal (Theorem 4.1).

One may refer to [2], [4] and [5] for the theory of Kuramochi boundary, including the notions of full-harmonic and full-superharmonic functions, those of potential type, Kuramochi kernel (denoted by N in [4], [5] and by $\tilde{g}$ in [2]), minimal points and non-minimal points. To apply the general theory, we take the domain $\Omega = \hat{R}^d - F$ (instead of $R^d - F$), where $\hat{R}^d$ is the one point compactification of R^d. Ω is a space of type $\mathfrak{S}$ in the sense of Brelot-Choquet. Let B be the unit ball $\{x; |x| < 1\}$ in R^d and suppose F is contained in B. Then $K_0 = \hat{R}^d - B$ is a compact set in Ω. Thus we can consider full-superharmonic functions on $\Omega_0 = \Omega - K_0 = B - F$ relative to Ω. The set of all harmonic full-superharmonic functions of potential type on Ω_0 will be denoted by $\mathcal{P}_b \equiv \mathcal{P}_b(\Omega_0)$ (cf. [4]). We remark here that any $u \in \mathcal{P}_b$ vanishes on $S = \{x; |x| = 1\}$, i.e., u is continuous if it is extended by 0 on S.

For a subset A in R^d, let $\bar{A}$ and ∂A be the closure and the boundary (in R^d) of A, respectively. If $A \subset P$, let $\partial' A$ be the boundary of A relative to the $(d-1)$-dimensional spacc P.