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Distinguished Normal Operators on Open Riemann Surfaces

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Introduction

Given a Riemann surface W, let $\mathcal{V}$ be the collection of open sets of W whose relative boundary consists of a finite number of closed analytic curves. For $V \in \mathcal{V}$ principal operators L_{0V} and $(P)L_{1V}$ were introduced by L. Sario (see [1]) and both share the property:

$$D_V(L_V f, L_V g) = \int_{\partial V} f(dL_V g)^*.$$

In this paper normal operator with this property will be called distinguished. We consider a system $\boldsymbol{L} = \{L_V\}_{V \in \boldsymbol{\mathcal{V}}}$ of distinguished normal operators L_V defined with respect to V. The system $\boldsymbol{L}$ is said to be consistent if the following condition is fulfilled:

$$L_{V_2}(L_{V_1} f) = L_{V_1} f$$

for any $V_1 \supset V_2$ and any continuous function f on ∂V_1.

Consider the Kerékjártó-Stoilow compactification W^* of W and denote the boundary by $\beta(W) = W^* - W$. Partition $\beta(W)$ into two disjoint sets α and γ where α is non-empty closed. The purpose of this paper is to investigate the following boundary value problems:

Suppose that the closure of $W_0 \in \mathcal{V}$ in W^* contains α and that f is continuously differentiable in W_0 and has $D_{W_0}(f) < \infty$. Then is there uniquely a function H_f satisfying the following conditions?

(I) H_f is harmonic in W and has $D_W(H_f) < \infty$,

(II) $H_f = L_V(H_f)$ for any $V \in \mathcal{V}$ such that the intersection of $\beta(W)$ with the closure of V is contained in γ,

(III) $\lim_{\substack{z \to \alpha \\ z \in \tau}} H_f(z) = \lim_{\substack{z \to \alpha \\ z \in \tau}} f(z)$ for almost all curves τ where each τ is a locally rectifiable curve in W, starting from a point of W and tending to α.

Roughly speaking, a solution H_f is to have $\boldsymbol{L}$-behavior on γ and assume given boundary values f on α.

We know by M. Ohtsuka [7], [9] that for the system $\boldsymbol{L}_0 = \{L_{0V}\}_{V \in \boldsymbol{\mathcal{V}}}$ we have the existence and uniqueness of H_f. We shall show that if the set α is