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Geometrical Association Schemes and Fractional Factorial Designs

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1. Summary

In this paper an attempt is made to throw light on the algebraic structure of symmetrical s^{k-p}-fractional factorial designs, where s is not necessary 2 but a prime power. For such purpose a geometrical factorial association scheme of PG$(k-1, s)$-type and the corresponding s^{k-p}-fractional factorial association scheme are introduced in sections 2 and 3 respectively. The corresponding association algebras $\mathfrak{A}(\mathrm{PG}(k-1, s))$ and $\mathfrak{A}(s^{k-p}-\mathrm{Fr})$ are also introduced there.

Mutually orthogonal idempotents of those algebras are given in section 4. The notion of fractionally similar mapping is introduced in section 5 and the relationship between $\mathfrak{A}(\mathrm{PG}(k-1, s))$ and $\mathfrak{A}(s^{k-p}-\mathrm{Fr})$ is investigated there. A general definition of the classical notion of aliases is given in section 6. Blocking of the fractional factorial designs is discussed in section 7 in relation to the notion of partial confounding and the pseudo-block factors.

The following notation is used throughout this paper:

I_n: The unit matrix of order n.

G_n: An $n\times n$ matrix whose elements are all unity.

A': Transpose of a matrix A.

$A\otimes B$: Kronecker product of the matrices $A=||a_{ij}||$ and B, i.e., $A\otimes B=||a_{ij}B||$.

$[A_i; i=1, \cdots, m]$: An algebra generated by the linear closure of those matrices indicated in the [ ].

GF(s): A finite field consists of s $(=q^u)$ elements, where q is a prime integer and u is a positive integer. An element a in GF(s) is represented by the coordinate representation or polynomial representation, i.e., $a=<a^{(1)}, \cdots, a^{(u)}>$ where $a^{(i)}$ is an element of GF(q), $i=1, 2,\cdots, u$.

EG(k, s): A k-dimensional Euclidean space over GF(s).

PG$(k-1, s)$: A $k-1$-dimensional projective space over GF(s).

$\mathfrak{P}(A)$: A subspace of PG$(k-1, s)$ generated by the linear closure of column vectors of a matrix A.

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