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The Method of Orthogonal Decomposition for Differentials on Open Riemann Surfaces

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Introduction

In the investigation of harmonic differentials on open Riemann surfaces, L. Ahlfors introduced the method of orthogonal decomposition and proved its effectiveness. In reality, it is by this method that he established the existence and uniqueness of a harmonic differential which has preassigned singularities and periods and which is subject to a certain prescribed boundary behavior.

In the classical case of closed Riemann surfaces, one of the main problems is to construct harmonic differentials with given periods and singularities. When we try to generalize the classical results to open surfaces in a non-trivial manner, it becomes necessary to add some restrictive conditions. Our restrictions will not be imposed on the surfaces, but merely on the differentials that are brought under consideration. In fact, it seems natural to make restrictions on differentials so that they behave mildly near the ideal boundary.

L. Ahlfors introduced the following mode of boundary behavior:

"A harmonic differential ω whose only singularities are harmonic poles is said to be *distinguished* if

(1) there exist differentials $\omega_{hm} \in \Gamma_{hm}$, $\omega_{e0} \in \Gamma_{e0} \cap \Gamma^1$ such that $\omega = \omega_{hm} + \omega_{e0}$ outside of a compact set,

(2) ω^* has vanishing periods along all dividing cycles which lie outside of a sufficiently large compact set."

On the other hand, in order to describe the boundary behavior of harmonic functions, L. Sario introduced the linear operators $(P)L_1$ and L_0, which he called *principal operators*. He established the existence and uniqueness of a harmonic function which has preassigned singularities and the boundary behavior described by one of principal operators.

In L. Ahlfors and L. Sario [4], the above two methods, namely *the method of orthogonal decomposition* and *the method of linear operators* are described quite separately, and the relation between them is not touched. In this paper we shall show that the former method yields also the result obtained by the latter as stated above.

In order to prescribe boundary behavior of harmonic functions and differentials, we choose *an arbitrary closed linear subspace* Γ_χ *of* Γ_{he}. The proofs