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Roots of Scalar Operator-valued Analytic Functions and their Functional Calculus

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Introduction

Let X be a Banach space, T a linear bounded operator acting in X and f an analytic complex function defined in a neighborhood of $\sigma(T)$. Let us suppose also that f is non-constant in each connected component of its domain of definition which intersects $\sigma(T)$.

In this paper we study the spectral properties of T if $f(T)$ is a spectral operator of scalar type. The example of Stampfli (see [18]) shows that in general T is not a scalar operator.

We shall prove that T is a Φ-scalar operator in the sense of [15], where Φ is a suitable basic algebra.

1. Preliminaries

Throughout the paper we shall use the following basic notation and conventions:

N: the set of all natural numbers.

Λ: the set of all complex numbers.

$\sigma'=\Lambda-\sigma$ for $\sigma\subset\Lambda$.

$C(K, r)=\{\lambda\in\Lambda;\ \text{dist}(\lambda, K)\leqq r\}$, where $K(\subset\Lambda)$ is compact and $r\geqq 0$.

$\mathfrak{F}(K)$: the set of all analytic complex functions whose domains of definition are open sets containing K, where K is a compact subset of Λ.

$\mathfrak{X}$: a Banach space over the complex field Λ.

$\mathfrak{L}(\mathfrak{X})$: the algebra of all linear bounded operators acting in $\mathfrak{X}$.

I: the unity of $\mathfrak{L}(\mathfrak{X})$.

$\sigma(T)$: the spectrum of $T\in\mathfrak{L}(\mathfrak{X})$.

Let $T\in\mathfrak{L}(\mathfrak{X})$ and $f\in\mathfrak{F}(\sigma(T))$. Then $f(T)=\frac{1}{2\pi i}\int_\Gamma f(\lambda)R(\lambda;T)d\lambda$, where

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