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Boundary Value Problems for the Equation $\Delta u-qu=0$ with respect to an Ideal Boundary

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Introduction

It is by now a classical result that linear boundary value problems for a second order elliptic linear partial differential equation with sufficiently smooth coefficients are uniquely solvable if boundary conditions and the boundary itself are sufficiently regular. While discussions for equations with non-smooth coefficients have been tried by many people, not much investigations of boundary value problems for non-smooth boundary have been made except for the Dirichlet problem.

As to the Dirichlet problem for the Laplace equation $\Delta u=0$, there is the method of Perron-Brelot (see [3], [7], etc.), which is also applied to more general equations (see e.g. [1], [4], [13]). For boundary value problems other than the Dirichlet problem, there appears the notion of normal derivatives, which is originally defined only with respect to a smooth boundary. Therefore, as long as we try to consider problems like Neumann problem and the third boundary value problem with respect to a non-smooth boundary, it is necessary to generalize the notion of normal derivatives in some way. This has been done by L. Doob [11] with respect to the Martin boundary, by C. Constantinescu and A. Cornea [7] with respect to the Kuramochi boundary and by the author [20] with respect to a general resolutive ideal boundary. In these works, linear boundary value problems involving normal derivatives are treated for the Laplace equation, i.e., for harmonic functions. In the present treatise, we apply the techniques developed in these works to the equation $\Delta u-qu=0$ $(q\geqq 0)$ and consider general linear boundary value problems with respect to a general ideal boundary.

We shall take a locally Euclidean space as the base space on which the equation is considered. It may be possible, however, to extend our theory to more general elliptic partial differential equations considered on a C^∞-manifold (cf. [12], [15], [16]). In fact, a locally Euclidean space is a special kind of C^∞-manifold and our theory may suggest how it is extended to a theory on a C^∞-manifold. Also, we can justify the restriction to the equation $\Delta u-qu=0$ by noting that this is the canonical form of self-adjoint equations (cf. [12], [14]).

This paper consists of the following six chapters: