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Integral Domains which are Almost Krull

Elbert M. Pirtle, Jr.

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1. Introduction.

In [1] Gilmer introduced the notion of almost-Dedekind domain. Every Dedekind domain is almost-Dedekind (AD) and AD-domains in general have many of the properties of Dedekind domains. Every Dedekind domain is a Krull domain in which proper nonzero prime ideals are maximal. Hence it seems natural to look for the proper generalization of almost-Dedekind domains to almost-Krull domains.

2. Definition and general properties.

In what follows, R denotes a commutative integral domain with identity and K denotes the quotient field of R. Proper prime ideals of R are nonzero prime ideals which are not equal to R. The notation is that found in [3] and [4].

We state the following definition from [1].

Definition 2.1. *R is AD iff R_M is a Dedekind domain for each maximal ideal M of R.*

It follows that proper prime ideals in an AD-domain are maximal. Now if R is AD and X is an indeterminate, then $R[X]$ is not AD. However, we do have the following proposition which motivates the definition of almost-Krull domain.

Proposition 2.2. *Let R be an AD-domain and let X be an indeterminate. Then for every proper prime ideal P of $R[X]$, the ring $R[X]_P$ is a Krull domain.*

Proof. Put $Q=P\cap R$ and let $M=R[X]-P$. Let $M_1=R-Q$, $M_2=M-M_1$. Now Q is a prime ideal of R so that M_1 is a multiplicative system in R and hence in $R[X]$. M_2 is the set of nonconstant polynomials in M and hence M_2 is also a multiplicative system in $R[X]$. It follows that $(R[X])_P=(R_{M_1}[X])_{M_2}$. Since R is AD, R_{M_1} is either a field or a Dedekind domain (If $P\cap R=(0)$ then $R_{M_1}=K$.). Thus $R_{M_1}[X]$ is a Krull domain. Since M_2 is a multiplicative system in $R_{M_1}[X]$, $(R_{M_1}[X])_{M_2}$ is a Krull domain.

The above proposition suggests the following definition.