J. Sci. Hiroshima Univ. Ser. A-I
32 (1968) 357-370

Two-way Classification Designs with Unequal Cell Frequencies

Sumiyasu Yamamoto and Yasunori Fujikoshi

(Received September 20, 1968)

1. Introduction and Summary

In a two-way classification design on two factors, say A and B, we apply each factor on varying levels to various experimental units. We assume that this application yields for each unit a quantity which we call the yield of this unit. We denote by $\eta(I, J)$ the mean value of the yield obtained when the factor A is applied at level I and the factor B at level J. These levels may be qualitative or quantitative and could assume discrete or continuous values. Usually they are chosen deterministically by the experimenter. In some cases, however, they are selected randomly according to a probability scheme. Even when the levels vary continuously, the experimenter can calibrate or can group them into a finite number of discrete values. We, therefore, assume that I and J can take the values $1, 2, \cdots, r$ and $1, 2, \cdots, s$, respectively.

The object of a two-way classification design is to make some inferences on the behavior of the mean yield function $\eta(I, J)$. For such purpose, the function $\eta(I, J)$ is usually broken up into a general mean μ, a main effect $\alpha(I)$ of the factor A, a main effect $\beta(J)$ of the factor B, and an interaction effect $\gamma(I, J)$ ascribed to the combination of level I of the factor A with level J of the factor B, i.e.,

$$\eta(I, J)=\mu+\alpha(I)+\beta(J)+\gamma(I, J). \tag{1.1}$$

If nothing more is stated about the decomposition, these components of the decomposition are not uniquely defined. It is for this reason to impose some constraints among these components.

In order to seek for a set of reasonable and intutively acceptable constraints, we introduce a non-negative weight function $W(I, J)$ associated to a pattern of the yield function $\eta(I, J)$. A purpose of introducing such a weight function is to develop a unified treatment of the identification problem in the decomposition (1.1) of the yield function.

The weight function $W(I, J)$ might be considered as an apriori probability function over the combination of the levels of the experiment in the wide sense (the total mass of the distribution of $W(I, J)$ may not be necessarily unity).

Upon introducing the weight function $W(I, J)$, the general mean μ is defined as an overall expectation of $\eta(I, J)$ with respect to the weight function