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Duality Theorems in Mathematical Programmings and Their Applications

Maretsugu YAMASAKI

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§ 1. Introduction

Extensions of the classical duality theorem in linear programming have been investigated by many authors. We shall be particularly concerned with the results of K. S. Kretschmer [11], K. Isii [9] and M. Ohtsuka [13]. In [11] the program was discussed in paired spaces and the dimension of the classical program was generalized from finite to infinite. A convex program in paired spaces was studied in [9]. The program in [13] is a potential-theoretic generalization of the classical one and is closely related to the theory of capacities.

In this paper, we shall investigate duality theorems and their applications. We reformulate the program in [13] in a form as in [9] and [11] and discuss Ohtsuka's duality theorem and sufficient conditions in it. Then we see that many results in [13] may be interpreted as special cases of those in [11]. We also obtain a new duality theorem in §5 which is a converse of Kretschmer's Theorem 3 and Dieter's strong duality theorem in [4]. Ohtsuka's sufficient conditions are given in a more general form in §6. Those conditions are free from potential theory. We also give some criteria which are different from Kretschmer's. In §§7-11, we indicate how the theory is applied to the potential-theoretic problems treated in [6], [8], [13], [14] and [16].

§ 2. Preliminaries

It is assumed that the reader is familiar with the theory of linear topological spaces as developed in [1] and [2]. The set of real numbers are denoted by R and the set of non-negative real numbers by R_0. Let X and Y be linear spaces (over R) and $((\ ,\ ))$ be a bilinear functional on $X \times Y$. We say that X and Y are paired under $((\ ,\ ))$ or that X and Y are in duality (relative to $((\ ,\ ))$) if the following two conditions are fulfilled:

(i) For any $x \neq 0$, there exists $y \in Y$ such that $((x, y)) \neq 0$.

(ii) For any $y \neq 0$, there exists $x \in X$ such that $((x, y)) \neq 0$.

If the linear spaces X and Y are in duality, the weak topology on X is denoted by $w(X, Y)$ and the Mackey topology on X is denoted by $s(X, Y)$. For a cone C in X, we set