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On the Decomposition of a Linearly Connected Manifold with Torsion.

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§ 1. Introduction

Let M be a differentiable manifold with a linear connection, and let Φ_x be the homogeneous holonomy group at a point $x \in M$. If the tangent vector space at x is decomposed into a direct sum of subspaces which are invariant under Φ_x, then by the parallel displacements along curves on M, parallel distributions are defined on M corresponding to those subspaces. If M is a Riemannian manifold and its connection is Riemannian, then by the de Rham decomposition theorem ([7] or [4] p. 185) the above parallel distributions are completely integrable and, at any point, M is locally isometric to the direct product of leaves through the point. Moreover, if M is simply connected and complete, it is globally isometric to the direct product of those leaves (see also [7] or [4] p. 192).

The above local and global decomposition theorems of de Rham are generalized to the case of pseudo-Riemannian manifold by H. Wu ([9]). On the other hand, in [2], S. Kashiwabara generalized the global decomposition theorem to the case of linearly connected manifold without torsion, under the assumption of local decomposability.

In the present paper, a linearly connected manifold with torsion will be treated and a condition of local decomposition will be given in terms of curvature and torsion (Theorem 1). Next, in §4, the results will be applied to a reductive homogeneous space with the canonical connection of the second kind, using the notion of algebra introduced by A. A. Sagle in [8].

Finally, in §5, we shall remark about the decomposition of a local loop with any point in M as its origin ([3]), corresponding to the local decomposition of the linearly connected manifold M.

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§ 2. Integrability of parallel distributions

Let (M, ∇) be a connected differentiable manifold with a linear connection, where ∇ means the covariant differentiation of the connection. The curvature tensor R and the torsion tensor S are defined by the formulas: