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A Generalization of Kuhn's Theorem for an Infinite Game

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An extensive n-person game is usually described in terms of a finite tree in an oriented plane. The game involves in its structure the mixed and behavior strategies of each player closely related to some specified information sets. Under the assumption that each move possesses at least two alternatives, H.W. Kuhn [1] proved the theorem that the game has perfect recall if and only if, given any mixed strategies $\mu_1, \mu_2, \cdots, \mu_n$, there may be associated with them behavior strategies $\beta_1, \beta_2, \cdots, \beta_n$, each β_i depending only on the corresponding mixed strategy μ_i, so that they give rise to the equalities,

$$H_i(\mu_1, \mu_2, \cdots, \mu_n)=H_i(\beta_1, \beta_2, \cdots, \beta_n),\ i=1, 2, \cdots, n,$$

where H_i stands for any expected pay-off to the player i.

In this note we shall generalize the game to an infinite game, and show that there remains still valid an analogue to Kuhn's theorem just referred to. However, the term "*perfect recall*" should be understood in a more general sense in order to remove the assumption cited above.

§ 1. An infinite extensive n-person game

We shall introduce an infinite extensive game with which we shall be concerned in this note. To this end, we consider an ordered set $(E, \leq)$ with the properties:

(1) *E has the least element* x_0.

(2) *For any* $x, y \in E$, *if there exists a* $z \in E$ *such that* $x \leq z$ *and* $y \leq z$, *then* $x \leq y$ *or* $y \leq x$.

If $x \leq y$, we say that x is a predecessor of y, and y is a successor of x. If $x < y$ and there is no element z: $x < z < y$, then we say that x is an immediate predecessor of y and y an immediate successor of x.

(3) *Every* $x \in E$ *except* x_0 *has a unique immediate predecessor which will be denoted by* $f(x)$.

(4) *Every* $x \in E$ *has an immediate successor, and the set* $f^{-1}(x)$ *is finite.*

(5) *For each* $x \in E$, *there is an integer* $m \geqq 0$ *such that* $f^m(x) = x_0$, *where* f^0 *denotes the identical mapping.*