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Asymptotic Behavior of Solutions of Parabolic Differential Equations with Unbounded Coefficients

Dedicated to Professor Tokui Satō on the occasion of his retirement

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Introduction

Let $x=(x_1, \dots, x_n)$ denote points in the real Euclidean n-space E^n and t denote points on the real line E^1. The distance of a point x of E^n to the origin is defined by $|x|=(\sum_{i=1}^{n} x_i^2)^{1/2}$.

Consider the Cauchy problem

$$\sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2}+(-k^2|x|^2+l)u-\frac{\partial u}{\partial t}=0 \text{ in } E^n\times(0, \infty),$$

$$u(x, 0)=M\exp(a|x|^2) \text{ on } E^n,$$

where $k>0$, l, a and M are constants. It is shown in [5] that if $2a<k$ the solution of this problem exists and is given explicitly by

$$u(x, t)=M\left(\frac{k}{k\cosh 2kt-2a\sinh 2kt)}\right)^{n/2}$$

$$\times\exp\left[\frac{k(2a\cosh 2kt-k\sinh 2kt)}{2(k\cosh 2kt-2a\sinh 2kt)}|x|^2+lt\right].$$

This formula shows that if $l-kn$ is negative, then $u(x, t)$ tends to zero as $t\to\infty$, the convergence being of exponential order and uniform with respect to $x\in E^n$.

The purpose of the present paper is to prove similar results for general second order parabolic equations with unbounded coefficients. In Section 1 we investigate under what conditions the solutions of

$$\text{(A)}\qquad \sum_{i,j=1}^{n} a_{ij}(x, t)\frac{\partial^2 u}{\partial x_i\partial x_j}+\sum_{i=1}^{n} b_i(x, t)\frac{\partial u}{\partial x_i}+c(x, t)u-\frac{\partial u}{\partial t}=0$$

with unbounded initial values decay exponentially to zero as $t\to\infty$. In Section 2 the results of Section 1 are extended to weakly coupled parabolic systems of the form

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